

Are there infinitely many decompositions of the nucleon spin ?

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Abstract

We discuss the uniqueness or non-uniqueness problem of the decomposition of the gluon field into the physical and pure-gauge components, which is the basis of the recently proposed two physically inequivalent gauge-invariant decompositions of the nucleon spin. By introducing a constant 4-vector n^μ , which amounts to specifying a Lorentz frame of reference, we explicitly construct a projection operator which projects out the physical component of the gluon, which is thought to provide us with a lowest-order expression of more rigorous gauge-invariant definition of the gluon spin operator given in a seemingly covariant form. This result is then used to demonstrate the gauge- and Lorentz-frame independence of the 1-loop evolution equation of the longitudinal quark and gluon spins in the nucleon. By drawing on all of these findings together with well-established knowledge from electrodynamics, we argue against the rapidly spreading view in the community that there are infinitely many decompositions of the nucleon spin.

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I. INTRODUCTION

Is a gauge-invariant complete decomposition of the nucleon spin possible ? It is a fundamentally important question of QCD as a color gauge theory. The reason is that the gauge-invariance is a *necessary condition of observability*. Unfortunately, this is quite a delicate problem, which is still under intense debate [1] -[34]. In a series of papers [16] - [19], we have established the fact that there are two physically inequivalent gauge-equivalent decompositions of the nucleon spin, which we call the decomposition (I) and (II). The decomposition (I) and (II) are respectively characterized by two different orbital angular momenta (OAMs) for both of quarks and gluons, i.e. the “dynamical” OAMs and the generalized “canonical” OAMs. We also clarified the fact that difference of the above two kinds of orbital angular momenta is characterized by a quantity which we call the “potential angular momentum”, the QED correspondent of which is nothing but the angular momentum carried by the electromagnetic field or potential playing a key role in the famous Feynman paradox of classical electrodynamics [16],[35]. The basic assumption for obtaining these two gauge-invariant decompositions of the nucleon spin is that the total gluon field can be decomposed into the two parts, i.e. the physical component and the pure-gauge component, as $A^\mu(x) = A_{phys}^\mu(x) + A_{pure}^\mu(x)$. In the course of deriving the above two gauge-invariant decompositions of the nucleon spin, these two components are supposed to obey the following general conditions, i.e. the pure-gauge condition for the pure-gauge component, $F_{pure}^{\mu\nu} \equiv \partial^\mu A_{pure}^\nu - \partial^\nu A_{pure}^\mu - i g [A_{pure}^\mu, A_{pure}^\nu] = 0$, supplemented with the homogeneous (or covariant) and inhomogeneous gauge transformation properties respectively for the physical and pure-gauge components of the gluon field under general gauge transformation of QCD.

A natural question is whether these general conditions are enough to uniquely fix the above decomposition. The answer is evidently No ! Note however that the above decomposition is proposed as a covariant generalization of Chen et al.’s decomposition given in a noncovariant form as $\mathbf{A}(x) = \mathbf{A}_{phys}(x) + \mathbf{A}_{pure}(x)$ [8],[9]. One must know the fact that, at least in the QED case, this decomposition is nothing new. It just corresponds to the standardly-known transverse-longitudinal decomposition of the 3-vector potential of the photon field, i.e. $\mathbf{A}(x) = \mathbf{A}_\perp(x) + \mathbf{A}_\parallel(x)$ satisfying the properties $\nabla \cdot \mathbf{A}_\perp = 0$ and $\nabla \times \mathbf{A}_\parallel = 0$ [36], [37]. It is a well-established fact that this decomposition is *unique* once

the Lorentz frame of reference is specified [37]. As we shall see later, a physically essential element here is the transversality condition $\nabla \cdot \mathbf{A}_\perp = 0$ for the transverse (or physical) component of \mathbf{A} given in a non-covariant form. Naturally, a certain substitute of this condition is necessary to uniquely fix the physical component of A_{phys}^μ in the above-mentioned decomposition given in a (seemingly) covariant form. This fundamental fact of gauge theory is missed out in the community, and conflicting views have rapidly spread around.

On the one hand, Lorcé claims that the above decomposition is not unique because of the presence of what-he-call the Stueckelberg symmetry, which alters both of A_{phys}^μ and A_{pure}^μ while keeping their sum unchanged [30],[31]. This misapprehension comes from the oversight of the importance of the transversality condition that should be imposed on the physical component. On the other, another argument against the uniqueness of the above-mentioned decomposition is advocated by Ji et al. [32]-[34]. According to them, the Chen decomposition is a gauge-invariant extension (GIE) of the Jaffe-Manohar decomposition based on the Coulomb gauge, while the Bashinsky-Jaffe decomposition is a GIE of the Jaffe-Manohar decomposition based on the light-cone gauge. They claim that, since the way of GIE with use of path-dependent Wilson line is not unique at all, there is no need that the above two decompositions give the same physical predictions. This made Ji reopen his longstanding claim that the gluon spin ΔG in the nucleon is not a gauge-invariant quantity in a *true* or *traditional* sense, although it is a measurable quantity in polarized deep-inelastic scatterings [38],[39]. One should recognize a self-contradiction inherent in this claim. In fact, first remember the fundamental proposition of physics, which states that “Observables must be gauge-invariant.” The contraposition of this proposition (note that it is always correct if the original proposition is correct) is gGauge-variant quantities cannot be observables”. This dictates that, if ΔG is claimed to be observable, it must be gauge-invariant also in the *traditional* sense.

In view of the above-explained frustrated status, we believe it urgent to correct widespread misunderstanding on the meaning of *true* or *traditional* gauge-invariance in the problem of nucleon spin decomposition. The paper is then organized as follows. In sect.II, we first clarify the fact that, at least in the case of abelian gauge theory, the decomposition of the gauge field into the physical and pure-gauge component is nothing but the well-known transverse-longitudinal decomposition as far as we are working in a prescribed Lorentz frame. Next, we show that, by introducing a constant 4-vector n^μ , which amounts to specifying the

Lorentz-frame of reference, one can make this decomposition in a seemingly covariant form, although at the lowest order in the gauge coupling constant in the case of nonabelian gauge theory. This in turn enables us to write down a lowest-order expression of the gauge-invariant gluon spin operator usable in a desired Lorentz frame. Next in sect.III, based on the gluon spin operator derived above, we investigate the 1-loop anomalous dimension matrix of the quark and gluon spins, to verify the gauge- and Lorentz-frame independence of the evolution equation. In Sect.IV, we shall discuss some unsettled issues of the gauge-invariant-extension approach with use of the Wilson line. Concluding remarks will then be given in sect.V.

II. PROJECTING OUT PHYSICAL COMPONENT OF GLUON FIELD

In a series of papers [16] -[19], we have shown that there are two physically inequivalent decompositions of the nucleon spin, which we call the decomposition (I) and (II). The QCD angular momentum tensor in the decomposition (I) is given as follows :

$$M^{\mu\nu\lambda} = M_{q-spin}^{\mu\nu\lambda} + M_{q-OAM}^{\mu\nu\lambda} + M_{G-spin}^{\mu\nu\lambda} + M_{G-OAM}^{\mu\nu\lambda} + M_{boost}^{\mu\nu\lambda}, \quad (1)$$

with

$$M_{q-spin}^{\mu\nu\lambda} = \frac{1}{2} \epsilon^{\mu\nu\lambda\sigma} \bar{\psi} \gamma_\sigma \gamma_5 \psi, \quad (2)$$

$$M_{q-OAM}^{\mu\nu\lambda} = \bar{\psi} \gamma^\mu (x^\nu i D^\lambda - x^\lambda i D^\nu) \psi \quad (3)$$

$$M_{G-spin}^{\mu\nu\lambda} = 2 \text{Tr} [F^{\mu\lambda} A_{phys}^\nu - F^{\mu\nu} A_{phys}^\lambda], \quad (4)$$

$$M_{G-OAM}^{\mu\nu\lambda} = -2 \text{Tr} [F^{\mu\alpha} (x^\nu D_{pure}^\lambda - x^\lambda D_{pure}^\nu) A_\alpha^{phys}], \\ + 2 \text{Tr} [(D_\alpha F^{\alpha\mu}) (x^\nu A_{phys}^\lambda - x^\lambda A_{phys}^\nu)], \quad (5)$$

and

$$M_{boost}^{\mu\nu\lambda} = -\frac{1}{2} \text{Tr} F^2 (x^\nu g^{\mu\lambda} - x^\lambda g^{\mu\nu}). \quad (6)$$

On the other hand, the QCD angular momentum tensor in the decomposition (II) is given as follows :

$$M^{\mu\nu\lambda} = M_{q-spin}'^{\mu\nu\lambda} + M_{q-OAM}'^{\mu\nu\lambda} + M_{G-spin}'^{\mu\nu\lambda} + M_{G-OAM}'^{\mu\nu\lambda} + M_{boost}'^{\mu\nu\lambda}, \quad (7)$$

with

$$M'_{q-spin}{}^{\mu\nu\lambda} = M_{q-spin}{}^{\mu\nu\lambda}, \quad (8)$$

$$M'_{q-OAM}{}^{\mu\nu\lambda} = \bar{\psi} \gamma^\mu (x^\nu i D_{pure}^\lambda - x^\lambda i D_{pure}^\nu) \psi \quad (9)$$

$$M'_{G-spin}{}^{\mu\nu\lambda} = M_{G-spin}{}^{\mu\nu\lambda}, \quad (10)$$

$$M'_{G-OAM}{}^{\mu\nu\lambda} = -2 \text{Tr} [F^{\mu\alpha} (x^\nu D_{pure}^\lambda - x^\lambda D_{pure}^\nu) A_\alpha^{phys}], \quad (11)$$

$$M'_{boost}{}^{\mu\nu\lambda} = M^{\mu\nu\lambda}. \quad (12)$$

In these two decompositions, the quark and gluon intrinsic spin parts are just common, and the difference lies only in the orbital parts. The difference is given as follows :

$$\begin{aligned} M_{q-OAM}{}^{\mu\nu\lambda} - M'_{q-OAM}{}^{\mu\nu\lambda} &= - \left(M_{G-OAM}{}^{\mu\nu\lambda} - M'_{G-OAM}{}^{\mu\nu\lambda} \right) \\ &= 2 \text{Tr} [(D_\alpha F^{\alpha\mu}) (x^\nu A_{phys}^\lambda - x^\lambda A_{phys}^\nu)]. \end{aligned} \quad (13)$$

The quantity characterizing the difference between the two kinds of orbital angular momenta of quarks and gluons, i.e. the quantity appearing in the r.h.s. of the above relation, is a covariant generalization of the following quantity

$$\mathbf{L}_{pot} = \int \rho^a (\mathbf{r} \times \mathbf{A}^a) d^3r \quad (14)$$

which we called the *potential angular momentum* in [16]. The reason is that this just corresponds to the angular momentum carried by the electromagnetic field or potential appearing in famous Feynman's paradox of classical electrodynamics [35]. (For an interesting phenomenological consequence of the two physically inequivalent decompositions of the nucleon spin, we refer to the references [40] -[45].)

The whole argument above is based on the decomposition of the gluon field A^μ into the physical component and the pure-gauge component as

$$A^\mu = A_{phys}^\mu + A_{pure}^\mu, \quad (15)$$

satisfying the following general conditions, i.e. the pure-gauge condition for A_{pure}^μ

$$F_{pure}^{\mu\nu} \equiv \partial^\mu A_{pure}^\nu - \partial^\nu A_{pure}^\mu - i g [A_{pure}^\mu, A_{pure}^\nu] = 0, \quad (16)$$

supplemented with the gauge-transformation properties for A_{phys}^μ and A_{pure}^μ

$$A_{phys}^\mu(x) \rightarrow U(x) A_{phys}^\mu(x) U^\dagger(x), \quad (17)$$

$$A_{pure}^\mu(x) \rightarrow U(x) \left(A_{pure}^\mu(x) + \frac{i}{g} \partial^\mu \right) U^\dagger(x), \quad (18)$$

under an arbitrary gauge transformation $U(x)$ of QCD.

In recent papers [30],[31], Lorce criticized that the pure-gauge condition $F_{pure}^{\mu\nu} = 0$ is insufficient to uniquely determine the decomposition $A^\mu = A_{phys}^\mu + A_{pure}^\mu$. According to him, there exists a hidden symmetry, which he calls a Stueckelberg symmetry. In the simpler case of abelian gauge theory, the proposed Stueckelberg transformation is given by

$$A_{phys}^\mu(x) \rightarrow A_{phys,g}^\mu(x) = A_{phys}^\mu(x) - \partial^\mu C(x), \quad (19)$$

$$A_{pure}^\mu(x) \rightarrow A_{pure,g}^\mu(x) = A_{pure}^\mu(x) + \partial^\mu C(x), \quad (20)$$

with $C(x)$ being an arbitrary function of space and time. Certainly, this transformation changes both of A_{phys}^μ and A_{pure}^μ , but the sum of them is intact. It was then claimed that this hidden symmetry dictates the existence of infinitely many decompositions of the gauge field into the physical and pure-gauge components, thereby leading him to the conclusion that there are in principle *infinitely many decompositions* of the nucleon spin.

It is certainly true that the pure-gauge condition, together with the homogeneous and inhomogeneous transformation properties of A_{phys}^μ and A_{pure}^μ , are not sufficient to determine the decomposition $A^\mu = A_{phys}^\mu + A_{pure}^\mu$ uniquely. However, one should remember the original motivation of this decomposition. In the QED case with noncovariant treatment by Chen et al. [8],[9], this decomposition is nothing more than the standard decomposition of the vector potential \mathbf{A} of the photon field into the transverse and longitudinal components :

$$\mathbf{A} = \mathbf{A}_\perp + \mathbf{A}_\parallel, \quad (21)$$

where the transverse component and the longitudinal component are respectively required to obey divergence-free and irrotational conditions :

$$\nabla \cdot \mathbf{A}_\perp = 0, \quad \nabla \times \mathbf{A}_\parallel = 0. \quad (22)$$

As is well-known, these two components transform as follows,

$$\mathbf{A}_\perp(x) \rightarrow \mathbf{A}'_\perp(x) = \mathbf{A}_\perp(x), \quad (23)$$

$$\mathbf{A}_\parallel(x) \rightarrow \mathbf{A}'_\parallel(x) = \mathbf{A}_\parallel(x) - \nabla \Lambda(x). \quad (24)$$

under general abelian gauge transformations. This means that \mathbf{A}_\parallel carries unphysical gauge degrees of freedom, while \mathbf{A}_\perp is absolutely intact under gauge transformation. Besides, it is a well-established fact that this decomposition is *unique*, once the Lorentz-frame of reference

is specified [37]. (To be more precise, the uniqueness is guaranteed by a supplemental condition that \mathbf{A} falls off faster than $1/r^2$ at the spatial infinity, which is satisfied in usual circumstances that happen in the electromagnetism.) This uniqueness of the decomposition indicates that, in QED, there exists no Stueckelberg symmetry as suggested by Lorcé. In fact, within the above-mentioned noncovariant framework, the Stueckelberg transformation a la Lorcé reduces to

$$\mathbf{A}_\perp(x) \rightarrow \mathbf{A}_\perp^g(x) = \mathbf{A}_\perp(x) + \nabla C(x), \quad (25)$$

$$\mathbf{A}_\parallel(x) \rightarrow \mathbf{A}_\parallel^g(x) = \mathbf{A}_\parallel(x) - \nabla C(x). \quad (26)$$

One can convince that the transformed component $\mathbf{A}_\parallel^g(x)$ retains the irrotational property for the longitudinal component,

$$\nabla \times \mathbf{A}_\parallel^g = \nabla \times (\mathbf{A}_\parallel - \nabla C(x)) = \nabla \times \mathbf{A}_\parallel = 0. \quad (27)$$

(This is simply a reflection of the fact the standard gauge transformation for \mathbf{A}_\parallel keeps the magnetic field intact.) However, one finds that the transformed component $\mathbf{A}_\perp^g(x)$ does not satisfy the desired divergence-free (or transversality) condition $\nabla \cdot \mathbf{A}_\perp^g = 0$ any more, since

$$\nabla \cdot \mathbf{A}_\perp^g = \nabla \cdot (\mathbf{A}_\perp + \nabla C(x)) = \Delta C(x) \neq 0, \quad (28)$$

unless $\Delta C(x) = 0$. (As a matter of course, different from the Stueckelberg transformation, there is no such problem in the standard gauge transformation (23) and (24), because \mathbf{A}_\perp is intact under the general gauge transformation.) The condition $\Delta C(x) = 0$ means that $C(x)$ is a harmonic function in three spatial dimension. If it is required to vanish at the spatial infinity, it must be identically zero owing to the Helmholtz theorem. As is clear from the discussion above, the Stueckelberg-like transformation does not generally preserve the transversality condition of the transverse or physical component of \mathbf{A} . In other words, the Stueckelberg symmetry does not actually exist and/or it has nothing to do with a physical symmetry of QED. Let us repeat again the well-founded fact in QED. The transverse-longitudinal decomposition is unique once the Lorentz-frame of reference is fixed.

Still, a bothersome problem here is that the transverse-longitudinal decomposition is not a relativistically invariant manipulation. A vector field that appears transverse in a certain Lorentz frame is not necessarily transverse in another Lorentz frame. An immediate question is then what meaning one can give to the seemingly covariant decomposition of

the gauge field like $A^\mu = A_{phys}^\mu + A_{pure}^\mu$. Putting it in another way, in view of the fact that the transverse-longitudinal decomposition can be made only at the sacrifice of breaking the Lorentz-covariance, how can we get an explicit form of this decomposition, which is usable in a desired Lorentz frame ? Before answering this nontrivial question, we want to make some general remarks on the treatment of gauge theories. In a covariant treatment of gauge theories, we start with the gauge field A^μ with four components ($\mu = 0, 1, 2, 3$). We however know that the massless gauge field has only two independent dynamical degrees of freedom, i.e., two transverse components, say A^1 and A^2 . The other two components, i.e. the scalar component A^0 and the longitudinal component A^3 , are not independent dynamical degrees of freedom. For quantizing a gauge theory, we need a procedure of gauge-fixing. A gauge-fixing procedure is essentially an operation, which eliminates the unphysical degrees of freedom so as to pick out the two transverse components. In this sense, the transverse-longitudinal decomposition and the gauge-fixing procedure are closely interrelated operations (one might say that they are nearly *synonymous*), although they are not absolutely identical operation as we shall see below.

We first point out that, in the absence of the matter fields which couple to the gauge field, the gauge-fixing procedure is basically unique, because in this case one can simply set both of A^0 and A^3 to be zero. This corresponds to the so-called radiation gauge [47]. (As a matter of course, the fully covariant treatment like the Feynman gauge or the Landau gauge retains both of A^0 and A^3 and handle the 4 component of A^μ on the equal footing although at the cost of introducing Hilbert space of indefinite metric.) However, for interacting theory, the situation is more complicated. In fact, this is the reason why there exist several independent gauge-fixing procedures, even if one restricts to a class of gauges called the “physical gauge”. The existence of plural forms of gauge-fixing procedure also indicates that the transverse-longitudinal decomposition may take several seemingly different appearances. Nevertheless, the fact is that a final answer for a truly gauge-invariant observables must be absolutely independent of the choice of gauge-fixing procedures, which implies that a similar can be anticipated for the transverse-longitudinal decomposition.

To find out a method for projecting out the transverse component of the gauge field, we start with a simpler problem of free photon. The massless free photon with momentum k is intrinsically transverse and therefore the photon states are described in terms of two polarization vectors $\varepsilon_\mu(\lambda = 1, k)$ and $\varepsilon_\mu(\lambda = 2, k)$ with $k^2 = 0$, $k^0 > 0$. These two polarization

vectors are orthogonal to each other and also orthogonal to the photon momentum k_μ [46] :

$$g^{\mu\nu} \varepsilon_\mu^*(\lambda, k) \varepsilon_\nu(\lambda', k) = -\delta^{\lambda\lambda'}, \quad (29)$$

$$k^\mu \varepsilon_\mu(\lambda, k) = 0. \quad (30)$$

The polarization vectors belonging to different gauges are related through

$$\varepsilon_\mu(\lambda, k) \rightarrow \varepsilon_\mu(\lambda, k) + k_\mu f(k), \quad (31)$$

where $f(k)$ is arbitrary functions of k describing the same photon states. Note that, for real photon with $k^2 = 0$, the orthogonality conditions (29) and (30) are not affected under gauge transformations. It is convenient to introduce a sort of density matrix $\tilde{P}_{\mu\nu}(k)$ by the equation

$$\tilde{P}_{\mu\nu}(k) \equiv - \sum_{\lambda=1}^2 \varepsilon_\mu(\lambda, k) \varepsilon_\nu^*(\lambda, k). \quad (32)$$

This is just a quantity appearing in unitary equations in scattering amplitudes of photons. This density matrix $\tilde{P}_{\mu\nu}(k)$ changes under gauge transformations of polarization vectors. Nevertheless, the following relations are intact :

$$\tilde{P}^{\mu\nu}(k) \tilde{P}_{\nu\rho}(k) = \tilde{P}^\mu{}_\rho(k), \quad (33)$$

$$\tilde{P}^\mu{}_\mu(k) = 2, \quad (34)$$

$$k^\mu \tilde{P}_{\mu\nu}(k) = 0. \quad (35)$$

As can be easily convinced, the above density matrix is nothing but the projection operator which projects out the physical (or transverse) component of the photon field. That is, one can introduce the physical photon field $\tilde{A}_{phys}^\mu(k)$ in the momentum space by the relation

$$\tilde{A}_{phys}^\mu(k) \equiv \tilde{P}^{\mu\nu}(k) \tilde{A}_\nu(k). \quad (36)$$

By construction, $\tilde{A}_{phys}^\mu(k)$ is intrinsically transverse. Our next task is to find a concrete form of $\tilde{P}^{\mu\nu}(k)$, which can be used in actual perturbative calculations. In view of the fact that the noncovariant transversality condition $\mathbf{k} \cdot \tilde{\mathbf{A}}_{phys}(\mathbf{k}) = 0$ is Lorentz-frame dependent, one may introduce an appropriate constant 4-vector n^μ , and then try to express $\tilde{P}^{\mu\nu}(k)$ in terms of k^μ , n^μ , and the metric tensor $g^{\mu\nu}$. A general form of $\tilde{P}^{\mu\nu}(k)$ would be given by

$$\tilde{P}^{\mu\nu}(k) = N [g^{\mu\nu} + A k^\mu k^\nu + B (k^\mu n^\nu + k^\nu n^\mu) + C n^\mu n^\nu]. \quad (37)$$

To determine the unknown coefficients, one may impose the following conditions :

$$k^\mu \tilde{P}_{\mu\nu}(k) = 0, \quad (38)$$

$$n^\mu \tilde{P}_{\mu\nu}(k) = 0. \quad (39)$$

$$\tilde{P}^\mu{}_\mu(k) = 2, \quad (40)$$

for arbitrary k^2 . We point out that the 2nd condition $n^\mu \tilde{P}_{\mu\nu}(k) = 0$ is equivalent to requiring the relation

$$n_\mu \tilde{A}^\mu(k) = 0, \quad (41)$$

which precisely coincides with the gauge-fixing condition in the “general axial gauge”. Note also that, once the first two conditions above hold, the relation $\tilde{P}^{\mu\nu}(k) \tilde{P}_{\nu\rho}(k) = \tilde{P}^\mu{}_\rho(k)$ is automatically satisfied.

First, by requiring (38) for arbitrary k^2 , we get two relations

$$C = -\frac{k^2}{k \cdot n} B, \quad B = -\frac{1}{k \cdot n} (1 + A k^2). \quad (42)$$

Next, by requiring (39), we obtain

$$A = -\frac{n^2}{k \cdot n} B, \quad B = -\frac{1}{k \cdot n} (1 + C n^2). \quad (43)$$

Combining these relations, we have

$$A = \frac{n^2}{(k \cdot n)^2 - k^2 n^2}, \quad B = -\frac{k \cdot n}{(k \cdot n)^2 - k^2 n^2}, \quad C = \frac{k^2}{(k \cdot n)^2 - k^2 n^2}. \quad (44)$$

Finally, the condition (40) determine the absolute normalization to be $N = 1$. This gives

$$\tilde{P}_{\mu\nu}(k) = g_{\mu\nu} + \frac{n^2 k_\mu k_\nu}{(k \cdot n)^2 - k^2 n^2} - \frac{k \cdot n (k_\mu n_\nu + k_\nu n_\mu)}{(k \cdot n)^2 - k^2 n^2} + \frac{k^2 n_\mu n_\nu}{(k \cdot n)^2 - k^2 n^2}. \quad (45)$$

So far, the constant 4-vector n^μ is completely arbitrary. Noteworthy here is several special choices. We first point out that, if one takes n^μ to be a time-like vector $n^\mu = (1, 0, 0, 0)$, this operator precisely coincides with the projection operator proposed by Lavelle and McMullan [48],[49], which projects out the physical component of the photon fields as

$$\tilde{A}_{phys}^\mu(k) = \tilde{P}^{\mu\nu}(k) \tilde{A}_\nu(k). \quad (46)$$

The corresponding physical photon propagator is given by

$$D_{phys}^{\mu\nu}(x-y) = \int \frac{d^4 k}{(2\pi)^4} \langle T(A_{phys}^\mu(x) A_{phys}^\nu(y)) \rangle e^{i k \cdot (x-y)}. \quad (47)$$

Since $n^2 = 1$, the corresponding lowest order photon propagator in the momentum space is given by

$$\begin{aligned}\tilde{D}_{phys}^{\mu\nu}(k) &= \frac{-i}{k^2 + i\varepsilon} \tilde{P}^{\mu\nu}(k) \\ &= \frac{-i}{k^2 + i\varepsilon} \left[g^{\mu\nu} + \frac{k^\mu k^\nu - k \cdot n (k^\mu n^\nu + k^\nu n^\mu) + k^2 n^\mu n^\nu}{(k \cdot n)^2 - k^2} \right],\end{aligned}\quad (48)$$

with $n^\mu = (1, 0, 0, 0)$. This is nothing but the free photon propagator in the radiation gauge [47]. It is important to recognize that it is not the photon propagator in the standard Coulomb gauge, because of the presence of the term proportional to $n^\mu n^\nu$. The latter is given by

$$\tilde{D}_{Coulomb}^{\mu\nu}(k) = \frac{-i}{k^2 + i\varepsilon} \left[g^{\mu\nu} + \frac{k^\mu k^\nu - k \cdot n (k^\mu n^\nu + k^\nu n^\mu)}{(k \cdot n)^2 - k^2} \right]. \quad (49)$$

The difference is that the latter contains the instantaneous Coulomb interaction as well.

As an interesting another choice, one may take n^μ to be a light-like vector satisfying $n^2 = 0$. In this case, $\tilde{P}^{\mu\nu}(k)$ in (45) reduces to

$$\tilde{P}^{\mu\nu}(k) = g^{\mu\nu} - \frac{k^\mu n^\nu + k^\nu n^\mu}{k \cdot n} + \frac{k^2 n^\mu n^\nu}{(k \cdot n)^2}. \quad (50)$$

The physical photon propagator corresponding to this choice of projection operator becomes

$$\tilde{D}_{phys}^{\mu\nu}(k) = \frac{-i}{k^2 + i\varepsilon} \left[g^{\mu\nu} - \frac{k^\mu n^\nu + k^\nu n^\mu}{k \cdot n} + \frac{k^2 n^\mu n^\nu}{(k \cdot n)^2} \right]. \quad (51)$$

This is just the photon (or gluon) propagator in the light-cone gauge, which satisfies the so-called double transversality conditions [50],[51], i.e.

$$k_\mu \tilde{D}^{\mu\nu}(k) = \tilde{D}^{\mu\nu}(k) k_\nu = 0 \quad (52)$$

and

$$n_\mu \tilde{D}^{\mu\nu}(k) = \tilde{D}^{\mu\nu}(k) n_\nu = 0. \quad (53)$$

We have seen that the introduction of the constant 4-vector n_μ enables us to project out the physical component of the photon field in a seemingly covariant form. The necessity of introducing the vector n_μ is a reflection of the fact that the transverse condition is intrinsically a Lorentz-frame dependent concept. Note however that $\tilde{P}^{\mu\nu}(k)$ is basically unique once n^ν is given, which fixes a Lorentz-frame in which the quantization of the gauge field is carried out [47]. This is not the end of the story, however. We are actually interested in

the gauge fields interacting with the matter field. (In the case of nonabelian gauge theory, there also exist self-interactions of the gauge fields.) The presence of interactions means the existence of off-mass-shell photons in addition to on-mass-shell photons. To impose all the conditions (38)-(40) for arbitrary k^2 would be too restrictive. We recall that, what is crucial for our whole theoretical framework is to project out the physical (or transverse) components of the gauge fields. We have also shown above that this operation is not necessarily equivalent to the gauge-fixing condition. This implies that imposing the gauge-fixing condition $n_\mu \tilde{P}^{\mu\nu}(k) = 0$ is not an absolute demand for projecting out the physical component. On the other hand, the condition $\tilde{P}^\mu{}_\mu(k) = 2$ should be retained, since it represents the fact that the independent dynamical degrees of freedom of the gauge fields are just two. Still, it would be enough that this condition holds only for the external or on-mass-shell photons. To sum up, for interacting theory of our practical interest, we propose to impose the following two conditions on the projection operator $\tilde{P}^{\mu\nu}(k)$, i.e.

$$k_\mu \tilde{P}^{\mu\nu}(k) = \tilde{P}^{\mu\nu}(k) k_\nu = 0, \quad \text{for arbitrary } k^2, \quad (54)$$

and

$$\tilde{P}^\mu{}_\mu(k) = 2, \quad \text{for } k^2 = 0. \quad (55)$$

As before, the 1st condition (54) gives

$$B = -\frac{1}{k \cdot n} (1 + A k^2), \quad (56)$$

$$C = -\frac{k^2}{k \cdot n} B = \frac{k^2}{(k \cdot n)^2} (1 + A k^2). \quad (57)$$

On the other hand, the 2nd condition (55) requires that, for $k^2 = 0$,

$$2 = \tilde{P}^\mu{}_\mu(k) = N \left\{ 2 + \frac{k^2}{k \cdot n} - k^2 \left[1 - \frac{k^2}{(k \cdot n)^2} \right] A \right\} = 2 N. \quad (58)$$

This gives $N = 1$. Note that A is completely arbitrary, since we are now requiring the condition $\tilde{P}^\mu{}_\mu(k) = 2$ only for external on-mass-shell photons with $k^2 = 0$. Undoubtedly, the simplest choice would be to set $A = 0$. This is a reasonable choice, because $k^\mu k^\nu$ term is purely longitudinal, so that there is no compelling reason to include it in $\tilde{P}^{\mu\nu}(k)$, which is a projection operator of the transverse component of the photon field.

In this way, we arrive at

$$\tilde{P}^{\mu\nu}(k) = g^{\mu\nu} - \frac{k^\mu n^\nu + k^\nu n^\mu}{k \cdot n} + \frac{k^2 n^\mu n^\nu}{(k \cdot n)^2}. \quad (59)$$

The corresponding physical photon propagator is given by

$$\tilde{D}_{phys}^{\mu\nu}(k) = \frac{-i}{k^2 + i\varepsilon} \left[g^{\mu\nu} - \frac{k^\mu n^\nu + k^\nu n^\mu}{k \cdot n} + \frac{k^2 n^\mu n^\nu}{(k \cdot n)^2} \right]. \quad (60)$$

As already mentioned, if one takes n^μ to be a light-like 4-vector, this just coincide with the photon (or gluon) propagator in the light-cone gauge with double transversality conditions [50],[51]. Note however that we do not assume here n^μ to be a light-like vector. We leave n^μ to be an *arbitrary* constant 4-vector, which can be time-like, light-like or space-like one. Note also that the physical photon propagator above for arbitrary n^μ is slightly different from the propagator in the general axial gauge given by

$$\tilde{D}_{axial}^{\mu\nu}(k) = \frac{-i}{k^2 + i\varepsilon} \left[g^{\mu\nu} - \frac{k^\mu n^\nu + k^\nu n^\mu}{k \cdot n} + \frac{n^2 k^\mu k^\nu}{(k \cdot n)^2} \right]. \quad (61)$$

The way of construction of the projection operator of the physical (or transverse) component of the photon field described above is not particular to the abelian gauge theory. It appears that the basic physics concept of transversality for massless vector field is shared also by the non-abelian gauge theory. We therefore assume that the physical component of the gluon field appearing in our definition of the gluon spin operator

$$M^{\mu\nu\lambda} = 2 \text{Tr} \left[F^{\mu\lambda} A_{phys}^\nu - F^{\mu\nu} A_{phys}^\lambda \right], \quad (62)$$

is given by the following formula

$$A_{phys}^\mu(x) = P^{\mu\nu}(x) A_\nu(x), \quad (63)$$

with

$$P^{\mu\nu}(x) = g^{\mu\nu} - \frac{\partial^\mu n^\nu + \partial^\nu n^\mu}{n \cdot \partial} + \frac{\square n^\mu n^\nu}{(n \cdot \partial)^2}. \quad (64)$$

A question is whether $A_{phys}^\mu(x)$ defined by (63) and (64) has a desired covariant transformation property (17). First, in the case of abelian gauge theory, one can easily confirm that this property is in fact satisfied. To see it, it is enough to consider an infinitesimal abelian gauge transformation $U(x) = e^{ig\omega(x)} \simeq 1 + ig\omega(x)$. The gauge variation of $A_\nu(x)$ in this case is

$$\delta A_\nu(x) \equiv U(x) \left(A_\nu(x) + \frac{i}{g} \partial_\nu \right) U^\dagger(x) - A_\nu(x) \simeq \partial_\nu \omega(x). \quad (65)$$

This means that the gauge variation of $A_{phys}^\mu(x)$ is given by

$$\delta A_{phys}^\mu(x) = P^{\mu\nu}(x) \delta A_\nu(x) = P^{\mu\nu}(x) \partial_\nu \omega(x) = 0. \quad (66)$$

Here, use has been made of the relation $P^{\mu\nu}(x) \partial_\nu = 0$, which is a coordinate space representation of the transversality condition $\tilde{P}^{\mu\nu}(k) k_\nu = 0$. We thus confirm that, in the abelian case, $A_{phys}^\mu(x)$ is in fact gauge-invariant.

Unfortunately, a similar does not hold in the nonabelian gauge theory. To see it, let us consider again an infinitesimal gauge transformation $U(x) = e^{ig\omega(x)} \simeq 1 + ig\omega(x)$ with the SU(3)-valued function $\omega(x) = \omega^a(x) T^a$. The gauge variation of $A_\nu(x)$ in this case is given by

$$\delta A_\nu(x) \simeq \partial_\nu \omega(x) - ig [A_\nu(x), \omega(x)] \equiv D_\nu \omega(x). \quad (67)$$

From this, we find that

$$\begin{aligned} \delta A_{phys}^\mu(x) &= P^{\mu\nu}(x) \delta A_\nu(x) = P^{\mu\nu}(x) \{ \partial_\nu \omega(x) - ig [A_\nu(x), \omega(x)] \} \\ &= -ig P^{\mu\nu}(x) [A_\nu(x), \omega(x)] \neq -g [A_{phys}^\mu(x), \omega(x)]. \end{aligned} \quad (68)$$

This means that the desired covariant transformation property of $A_{phys}^\mu(x)$ is not satisfied strictly. Clearly, the cause of this trouble can be traced back to the existence of nonlinear self-coupling of the gluon field. The fact is that, because of this self-coupling, there is no free gluon state even if we turn off the coupling with the color-charged quark field. We recall that the same sort of complexity already appeared in Chen et al.'s formulation given in a Lorentz-noncovariant framework. According to them, the nonabelian generalization of the transverse and longitudinal conditions $\nabla \cdot \mathbf{A}_\perp = 0$ and $\nabla \times \mathbf{A}_\parallel$ is given as

$$\nabla^k A_\perp^{ak} + g f^{abc} A_\parallel^{bk} A_\perp^{ck} = 0, \quad (69)$$

$$\nabla^j A_\parallel^{ak} - \nabla^k A_\parallel^{aj} + g f^{abc} A_\parallel^{bj} A_\parallel^{ck} = 0. \quad (70)$$

An explicit solution to these highly nonlinear constraints would be given only in a form of perturbation series in the gauge coupling constant g . The lowest order solution reduces to the transverse-longitudinal decomposition in the abelian case, which corresponds to free gluon. (We stress that, after all, the basic assumption of perturbative QCD is the perturbative existence of free quark and gluon states.) Since our primary concern in the present paper is the validity or invalidity of the perturbative gauge-invariance of the gluon spin in the nucleon, we shall adopt the following strategy. The physical (or transverse) component of the gluon field given by (63) and (64) is a lowest order solution to more rigorous defining equation of $A_{phys}^\mu(x)$ satisfying the covariant transformation property as well. The gluon spin operator, in which $A_{phys}^\mu(x)$ is replaced by this approximate solution is regarded as the lowest

order expression of more rigorous gluon spin operator, so that it is expected to be usable in the calculation of the corresponding anomalous dimensions at the 1-loop level. (Probably, more sophisticated treatment would be required already at the 2-loop level.) On the basis of this expression of the lowest-order gluon spin operator containing arbitrary 4-vector n^μ , which is thought to specify the Lorentz frame in which the transversality condition is given and also the quantization of the gauge field is carried out, we investigate in the next section the n^μ dependence or independence of the 1-loop evolution matrix of the quark and gluon spins in the nucleon.

III. GAUGE- AND FRAME-INDEPENDENCE OF THE EVOLUTION MATRIX FOR QUARK AND GLUON LONGITUDINAL SPINS IN THE NUCLEON

A primary question we want to address in this section is whether the gluon spin term appearing in the longitudinal nucleon spin sum rule is a gauge-invariant quantity in a true or traditional sense or whether it is a quantity that has a meaning only in the light-cone gauge or in the extension based on the light-cone gauge. As already pointed out, the choice of gauge and Lorentz-frame as well as the transverse-longitudinal decomposition are all intricately intertwined, one must be very careful. Although we naturally believe that the longitudinal nucleon spin decomposition can be made in a Lorentz-frame independent way, a practical problem is how we can show it in conformity with the decomposition of the gluon field into the physical (transverse) and pure-gauge (longitudinal) components. For answering this question, it is instructive to remember the basis of the (longitudinal) momentum sum rule of the nucleon. The momentum sum rule of the nucleon is based on the following covariant relation,

$$\langle Ps | T_{\mu\nu}(0) | Ps \rangle = 2 P_\mu P_\nu, \quad (71)$$

where $T_{\mu\nu}$ is the (symmetric) QCD energy momentum tensor, while $|Ps\rangle$ is a nucleon state with momentum P and spin s . A useful technique for obtaining the momentum sum rule is to introduce a light-like constant vector n^μ with $n^2 = 0$. By contracting (71) with n^μ and n^ν , we have

$$\frac{\langle Ps | n^\mu T_{\mu\nu}(0) n^\nu | Ps \rangle}{2 (P^+)^2} = 1, \quad (72)$$

which provides us with a convenient basis for obtaining a concrete form of the momentum sum rule of QCD. Since Eq.(71) itself is relativistically covariant, the above choice of n^μ

is not an only choice, however. With the choice of arbitrary constant four-vector n^μ with $n^2 \neq 0$, we would have more general relation,

$$\frac{\langle Ps | n^\mu T_{\mu\nu}(0) n^\nu - \frac{1}{4} n^2 T^\mu{}_\mu(0) | Ps \rangle}{2(P \cdot n)^2} = 1, \quad (73)$$

Here, since $n^2 \neq 0$, the subtraction of the trace term is obligatory.

Similarly, the starting point for obtaining the longitudinal nucleon spin sum rule is the following covariant relation :

$$\langle Ps | M^{\lambda\mu\nu}(0) | Ps \rangle = J_N \frac{P_\rho s_\sigma}{M_N^2} [2 P^\lambda \epsilon^{\nu\mu\rho\sigma} - P^\mu \epsilon^{\lambda\nu\rho\sigma} - P^\nu \epsilon^{\mu\lambda\rho\sigma}], \quad (74)$$

where $M^{\lambda\mu\nu}$ is the angular momentum tensor of QCD, while

$$P^2 = M_N^2, \quad s^2 = -M_N^2, \quad s \cdot P = 0, \quad (75)$$

with s_μ being a covariant spin-vector of the nucleon. Note that, without loss of generality, we can take as $P^\mu = (P^0, 0, 0, P^3)$ and $s^\mu = (P^3, 0, 0, P^0)$ with $P^0 = \sqrt{(P^3)^2 + M_N^2}$. The longitudinal nucleon spin sum rule can be obtained by setting $\mu = 1, \nu = 2$, which gives

$$\langle Ps | M^{\lambda 12}(0) | Ps \rangle = -2 J_N \frac{1}{M_N^2} P^\lambda \epsilon^{12\rho\sigma} P_\rho s_\sigma = 2 J_N P^\lambda. \quad (76)$$

Contracting this relation with an arbitrary constant 4-vector n_λ , we therefore arrive at the basis equation of the longitudinal nucleon spin sum rule [1] :

$$J_N = \frac{1}{2} = \frac{\langle Ps | n_\lambda M^{\lambda 12}(0) | Ps \rangle}{2(P \cdot n)}. \quad (77)$$

In the following, we shall confine to the intrinsic spin parts of quarks and gluons appearing in the nucleon spin decompositions :

$$M_{q-spin}^{\lambda\mu\nu} = \frac{1}{2} \epsilon^{\lambda\mu\nu\sigma} \bar{\psi} \gamma_\sigma \gamma_5 \psi, \quad (78)$$

$$M_{G-spin}^{\lambda\mu\nu} = 2 \text{Tr} [F^{\lambda\nu} A_{phys}^\mu - F^{\lambda\mu} A_{phys}^\nu]. \quad (79)$$

(We recall the fact that the quark and gluon intrinsic spin parts are just common in both of the decomposition (I) and (II).)

As discussed in [18], the gluon spin operator in general gauges consists of three parts as

$$M_{G-spin}^{\lambda 12} = 2 \text{Tr} [F^{\lambda 1} A_{phys}^2 - F^{\lambda 2} A_{phys}^1] = V_A + V_B + V_C, \quad (80)$$

where

$$V_A = (\partial^\lambda A_a^1) A_{a,phys}^2 - (\partial^\lambda A_a^2) A_{a,phys}^1, \quad (81)$$

$$V_B = -[(\partial^1 A_a^\lambda) A_{a,phys}^2 - (\partial^2 A_a^\lambda) A_{a,phys}^1], \quad (82)$$

$$V_C = g f_{abc} A_b^\lambda [A_c^1 A_{a,phys}^2 - A_c^2 A_{a,phys}^1]. \quad (83)$$

We recall that, in the light-cone gauge $A^+ = 0$ with $n^+ = 1, n^- = 0$, only the V_A term contribute to the gluon spin.

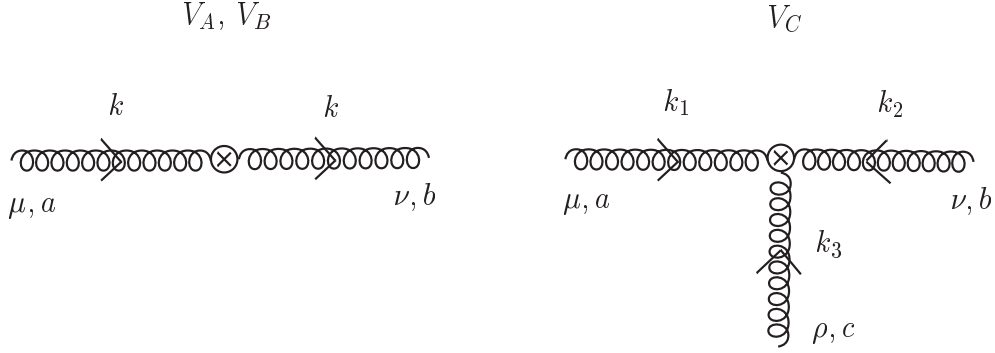


FIG. 1. Momentum space vertices for the gluon spin.

In view of the definition (80)-(83) of the gauge-invariant gluon spin operator, the momentum space vertex for the gluon spin can be expressed in the following forms supplemented with the diagrams illustrated in Fig.1 :

$$V_A = i k^\lambda (g^{\mu 1} \tilde{P}^{\nu 2}(k) - g^{\mu 2} \tilde{P}^{\nu 1}(k)) \delta_{ab} + (\mu \leftrightarrow \nu, a \leftrightarrow b, k \leftrightarrow -k), \quad (84)$$

$$V_B = -i g^{\lambda \mu} (k^1 \tilde{P}^{\nu 2}(k) - k^2 \tilde{P}^{\nu 1}(k)) \delta_{ab} + (\mu \leftrightarrow \nu, a \leftrightarrow b, k \leftrightarrow -k), \quad (85)$$

$$\begin{aligned} V_C = & g f_{abc} g^{\lambda \rho} (\tilde{P}^{\mu 1}(k_1) g^{\nu 2} - \tilde{P}^{\mu 2}(k_1) g^{\nu 1}) + (\mu \leftrightarrow \nu, a \leftrightarrow b, k_1 \leftrightarrow k_2), \\ & + g f_{abc} g^{\lambda \mu} (\tilde{P}^{\nu 1}(k_2) g^{\rho 2} - \tilde{P}^{\nu 2}(k_2) g^{\rho 1}) + (\nu \leftrightarrow \rho, b \leftrightarrow c, k_2 \leftrightarrow k_3), \\ & + g f_{abc} g^{\lambda \nu} (\tilde{P}^{\rho 1}(k_3) g^{\mu 2} - \tilde{P}^{\rho 2}(k_3) g^{\mu 1}) + (\rho \leftrightarrow \mu, c \leftrightarrow a, k_3 \leftrightarrow k_1). \end{aligned} \quad (86)$$

Now we are ready to investigate the anomalous dimension matrix for the longitudinal quark and gluon spins in the nucleon,

$$\Delta \gamma = \begin{pmatrix} \Delta \gamma_{qq} & \Delta \gamma_{qG} \\ \Delta \gamma_{Gq} & \Delta \gamma_{GG} \end{pmatrix}, \quad (87)$$

which controls the scale evolution of the quark and gluon spins. We start with the quark spin operator $M_{q-spin}^{\lambda 12}$, although there is no known problem in this part. The reason is that we want to convince the independence of the final result on the choice of the constant 4-vector n^μ , which specifies the Lorentz frame in which the transverse-longitudinal decomposition of the gluon spin is made. Besides, the same n^μ is thought to designate the Lorentz frame in which the gauge-fixing condition necessary for the quantization of the gauge field is given.

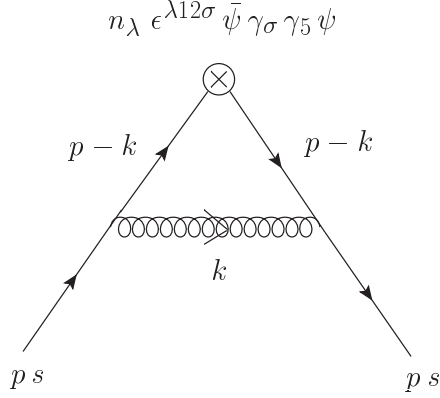


FIG. 2. The Feynman diagram contributing to $\Delta\gamma_{qq}$.

The anomalous dimension $\Delta\gamma_{qq}$ can be obtained by evaluating the matrix element of

$$2 n_\lambda M_{q-spin}^{\lambda 12} = n_\lambda \epsilon^{\lambda 12 \sigma} \bar{\psi} \gamma_\sigma \gamma_5 \psi, \quad (88)$$

in a longitudinally polarized quark state $|ps\rangle$ with $s = \pm 1$. The corresponding 1-loop diagram is shown in Fig.2. This gives

$$\begin{aligned} T_{qq} &= \frac{1}{2 p \cdot n} \int \frac{d^4 k}{(2\pi)^4} \bar{u}(ps) i g \gamma^\nu t^a \frac{i (\not{p} - \not{k})}{(p-k)^2 + i\epsilon} n_\lambda \epsilon^{\lambda 12 \sigma} \gamma_\sigma \gamma_5 \\ &\quad \times \frac{i (\not{p} - \not{k})}{(p-k)^2 + i\epsilon} i g \gamma^\mu t^b u(ps) \delta_{ab} D_{\mu\nu}(k), \end{aligned} \quad (89)$$

where

$$D_{\mu\nu}(k) = \frac{-i}{k^2 + i\epsilon} P_{\mu\nu}(k) \quad (90)$$

with

$$P_{\mu\nu}(k) \equiv P_{\mu\nu}^{axial}(k) = g_{\mu\nu} - \frac{k_\mu n_\nu + n_\mu k_\nu}{k \cdot n} + \frac{n^2 k_\mu k_\nu}{(k \cdot n)^2}. \quad (91)$$

Here, we use the gluon propagator in the general axial gauge. This is because we are working in a Lorentz frame specified by the constant 4-vector n^μ and because the quantization of the

gauge field is performed in this frame [47]. As is well-known, the gluon propagator in the general axial gauge contains a spurious simple pole and also a double pole. In the following, let us evaluate the contributions of the three terms in $P_{\mu\nu}^{axial}(k)$ separately. The calculation of the part containing $g_{\mu\nu}$ is straightforward. After some Dirac algebra, we get

$$T_{qq}(g_{\mu\nu}) = -i \frac{g^2 C_F}{2 p \cdot n} \times \left\{ -8 n_\alpha p_\beta \int \frac{d^4 k}{(2\pi)^4} \frac{k^\alpha k^\beta}{[(p-k)^2 + i\varepsilon]^2 (k^2 + i\varepsilon)} + 4 p \cdot n \int \frac{d^4 k}{(2\pi)^4} \frac{1}{[(p-k)^2 + i\varepsilon]^2} \right\}. \quad (92)$$

Using the standard dimensional regularization with $D \equiv 2\omega$ space-time dimension, the divergent parts of the necessary integral are given by

$$\text{div} \int \frac{d^4 k}{(2\pi)^4} \frac{k^\alpha k^\beta}{[(p-k)^2 + i\varepsilon]^2 (k^2 + i\varepsilon)} = \frac{1}{4} g^{\alpha\beta} \bar{I}, \quad (93)$$

$$\text{div} \int \frac{d^4 k}{(2\pi)^4} \frac{1}{[(p-k)^2 + i\varepsilon]^2} = \bar{I}, \quad (94)$$

where

$$\bar{I} = \frac{i\pi^2}{2-\omega}. \quad (95)$$

We therefore obtain

$$T_{qq}(g_{\mu\nu}) = \frac{\alpha_S}{2\pi} \frac{1}{2} C_F \frac{1}{2-\omega}. \quad (96)$$

Next, we evaluate the term containing a simple spurious pole $1/(k \cdot n)$. After some algebra, we get

$$T_{qq}(1/(k \cdot n)) = -i \frac{g^2 C_F}{2 p \cdot n} \left\{ -p \cdot n \int \frac{d^4 k}{(2\pi)^4} \frac{1}{[(p-k)^2 + i\varepsilon]^2} + n^2 p_\beta \int \frac{d^4 k}{(2\pi)^4} \frac{k^\beta}{[(p-k)^2 + i\varepsilon]^2 k \cdot n} \right\}. \quad (97)$$

Now we encounter a Feynman integral containing a spurious pole. A consistent method for handling such Feynman integrals was first proposed by Mandelstam [52] and independently by Leibbrandt [53] in the light-cone gauge corresponding to the choice $n^2 = 0$. It is given as

$$\frac{1}{k \cdot n} \rightarrow \frac{1}{[k \cdot n]} \equiv \lim_{\varepsilon \rightarrow 0} \frac{k \cdot n^*}{k \cdot n k \cdot n^* + i\varepsilon}, \quad (\varepsilon > 0) \quad (98)$$

where $n_\mu^* = (n_0, -\mathbf{n})$ is a dual 4-vector to the 4-vector $n_\mu = (n_0, \mathbf{n})$ with $n^2 = 0$ and $n^{*2} = 0$. (Practically, we can take as $n^\mu = (n^0, 0, 0, n^3)$ and $n^{*\mu} = (n^0, 0, 0, -n^3)$ without loss of generality.) Later, Gaigg et. al. showed that this prescription can be generalized to more

general case of $n^2 \neq 0$ and $n^{*2} \neq 0$ [54],[55]. (For review, see [56],[57].) In this generalized n_μ^* -prescription, the divergent part of the above integral is given by

$$\int d^2\omega k \frac{k^\beta}{[(p-k)^2 + i\varepsilon]^2 [k \cdot n]} = \frac{1}{D} \left(n^{*\beta} - \frac{n^{*2}}{n^* \cdot n + D} n^\beta \right) \bar{I}, \quad (99)$$

where D is defined by

$$D \equiv \sqrt{(n^* \cdot n)^2 - n^{*2} n^2}. \quad (100)$$

By using this result, the divergent part of $T_{qq}(1/(k \cdot n))$ becomes

$$T_{qq}(1/(k \cdot n)) = \frac{\alpha_S}{4\pi} C_F \frac{1}{2-\omega} \times \left\{ -4 + 2 \frac{n^2}{[p \cdot n]} \frac{1}{D} \left(p \cdot n^* - \frac{n^{*2}}{n^* \cdot n + D} p \cdot n \right) \right\}. \quad (101)$$

The contribution of the part containing a spurious double pole structure $1/(k \cdot n)^2$ can similarly be calculated. We get

$$T_{qq}(1/(k \cdot n)^2) = -i g^2 C_F n^2 \int \frac{d^4 k}{(2\pi)^4} \frac{k^2}{[(p-k)^2 + i\varepsilon]^2 (k \cdot n)^2}. \quad (102)$$

Using the generalized n_μ^* -prescription again, the divergent part of the relevant integral is given by

$$\text{div} \int \frac{d^4 k}{(2\pi)^4} \frac{k^2}{[(p-k)^2 + i\varepsilon]^2 [k \cdot n]^2} = \frac{2}{D} \frac{n^{*2}}{n^* \cdot n + D} \bar{I}. \quad (103)$$

We therefore obtain

$$T_{qq}(1/(k \cdot n)^2) = -2 \frac{\alpha_S}{4\pi} C_F \left(1 - \frac{n \cdot n^*}{D} \right) \frac{1}{2-\omega}. \quad (104)$$

Here, use has been made of the identity,

$$\frac{n^2 n^{*2}}{n^* \cdot n + D} = n \cdot n^* - D. \quad (105)$$

Summing up the three terms, we arrive at

$$\begin{aligned} T_{qq} = & -\frac{\alpha_S}{4\pi} C_F \frac{1}{2-\omega} \\ & -\frac{\alpha_S}{4\pi} 2 C_F \frac{1}{[p \cdot n]} \frac{1}{D} [p \cdot n n \cdot n^* - p \cdot n^* n^2] \frac{1}{2-\omega} \\ & -\frac{\alpha_S}{4\pi} 2 C_F \left(1 - \frac{n \cdot n^*}{D} \right) \frac{1}{2-\omega}. \end{aligned} \quad (106)$$

At this stage, it is instructive to consider several special choices of n^μ . The light-cone gauge choice corresponds to taking $n^0 = n^3 = 1/\sqrt{2}$. In this case, we have

$$p \cdot n = p^+, \quad p \cdot n^* = p^-, \quad n \cdot n^* = 1, \quad n^2 = 0, \quad (107)$$

and

$$D = 1, \quad (108)$$

so that we find that

$$T_{qq}(LC) = -\frac{\alpha_S}{2\pi} \frac{3}{2} C_F \frac{1}{2-\omega}. \quad (109)$$

This legitimately reproduces the answer first obtained by Ji, Tang and Hoodbhoy in the light-cone gauge [38].

Another interesting choice is the temporal gauge limit specified by $n^0 = 1$ and $n^3 = 0$. In this limit, we have

$$p \cdot n = p \cdot n^* = p^0, \quad n \cdot n^* = 1, \quad n^2 = 1 \quad (110)$$

and

$$D = 0. \quad (111)$$

We therefore find that the coefficients of $1/(2-\omega)$ in the 2nd and 3rd term of T_{qq} *diverge*. The temporal gauge limit is *singular* in this respect. However, for obtaining the anomalous dimension $\Delta\gamma_{qq}$, we must also take account of the self-energy insertion to the external quark lines. The contribution of these diagrams can easily be obtained by using the known result for the 1-loop quark self-energy in the general axial gauge. (See, for instance, [56]). We get

$$\begin{aligned} T_{qq}^{Self} = & \frac{\alpha_S}{4\pi} C_F \frac{1}{2-\omega} \\ & + \frac{\alpha_S}{4\pi} 2 C_F \frac{1}{[p \cdot n]} \frac{1}{D} [p \cdot n n \cdot n^* - p \cdot n^* n^2] \frac{1}{2-\omega} \\ & + \frac{\alpha_S}{4\pi} 2 C_F \left(1 - \frac{n \cdot n^*}{D}\right) \frac{1}{2-\omega}. \end{aligned} \quad (112)$$

As anticipated, this exactly cancels T_{qq} obtained above, thereby being led to the standardly-known answer, i.e.

$$\Delta\gamma_{qq} = 0. \quad (113)$$

It is important to recognize that this final result is obtained totally independently of the choice of the 4-vector n^μ .

The relevant Feynman diagram contributing to the anomalous dimension $\Delta\gamma_{qG}$ is illustrated in Fig.3. Since no internal gluon propagator appears in this diagram, we do not need to repeat the standard manipulation. One can easily verify that

$$\Delta\gamma_{qG} = 0. \quad (114)$$

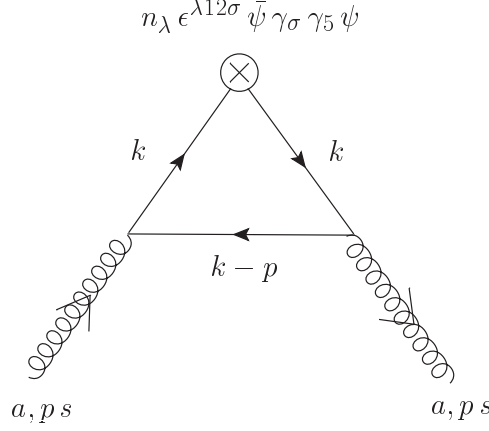


FIG. 3. The Feynman diagram contributing to $\Delta\gamma_{qG}$.

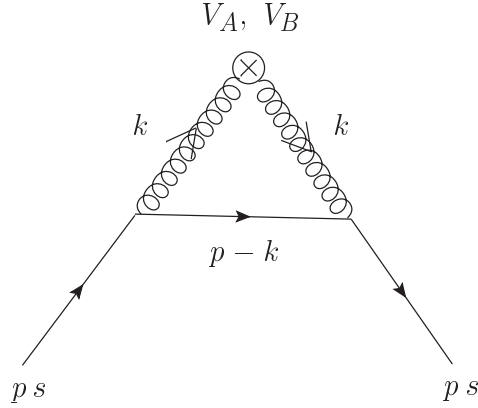


FIG. 4. The Feynman diagram contributing to $\Delta\gamma_{Gq}$.

Next, we turn to the anomalous dimension $\Delta\gamma_{Gq}$. The relevant 1-loop Feynman diagram is shown in Fig.4. The contribution of the vertex V_A in the gluon spin operator is given by

$$\begin{aligned}
T_{Gq}^A = & \frac{1}{2p \cdot n} \int \frac{d^4 k}{(2\pi)^4} \bar{u}(ps) i g \gamma^\nu t^d \frac{i(\not{p} - \not{k})}{(p - k)^2 + i\varepsilon} \delta^{de} \\
& \times i(k \cdot n) [g^{\mu 1} \tilde{P}^{\nu 2}(k) - g^{\mu 2} \tilde{P}^{\nu 1}(k)] \delta^{bc} i g \gamma^\mu t^e u(ps) \\
& \times \delta^{bd} D_{\mu\mu'}(k) \delta^{ce} D_{\nu\nu'}(k),
\end{aligned} \tag{115}$$

where $D_{\mu\mu'}$ and $D_{\nu\nu'}$ are gluon propagators excluding trivial color dependent parts. The point of the analysis below is the following. For a propagator in general places, we use the gluon propagator in the general axial gauge, i.e.

$$D_{\mu\nu}(k) \rightarrow \frac{-i}{k^2 + i\varepsilon} P_{\mu\nu}(k), \tag{116}$$

with

$$P_{\mu\nu}(k) \equiv P_{\mu\nu}^{axial}(k) = g_{\mu\nu} - \frac{k_\mu n_\nu + n_\mu k_\nu}{k \cdot n} + \frac{n^2 k_\mu k_\nu}{(k \cdot n)^2}, \quad (117)$$

in conformity with the fact that the quantization of the gauge field is carried out in the Lorentz frame specified by the 4-vector n_μ . On the other hand, if one of the Lorentz index of $D_{\mu\nu}(k)$ is contracted with the transverse projection operator $\tilde{P}_{\mu\alpha}(k)$ or $\tilde{P}_{\beta\nu}(k)$, we use the physical or transverse gluon propagator given by

$$D_{\mu\nu}(k) \rightarrow \frac{-i}{k^2 + i\varepsilon} \tilde{P}_{\mu\nu}(k), \quad (118)$$

with

$$\tilde{P}_{\mu\nu}(k) \equiv P_{\mu\nu}^{phys}(k) = g_{\mu\nu} - \frac{k_\mu n_\nu + n_\mu k_\nu}{k \cdot n} + \frac{k^2 n_\mu n_\nu}{(k \cdot n)^2}. \quad (119)$$

This gives

$$\begin{aligned} T_{Gq}^A &= - \frac{g^2 C_F}{2 p \cdot n} \int \frac{d^4 k}{(2\pi)^4} \frac{k \cdot n}{(k^2 + i\varepsilon)^2 [(k-p)^2 + i\varepsilon]} \\ &\quad \times \bar{u}(ps) \gamma^{\nu'} (\not{p} - \not{k}) \gamma^{\mu'} u(ps) (g^{\mu 1} g^{\nu 2} - g^{\mu 2} g^{\nu 1}) P_{\mu\mu'}(k) \tilde{P}_{\nu\nu'}(k) \\ &\quad - (\mu \leftrightarrow \nu) \\ &= i \frac{g^2 C_F}{p \cdot n} \int \frac{d^4 k}{(2\pi)^4} \frac{k \cdot n}{(k^2 + i\varepsilon)^2 [(k-p)^2 + i\varepsilon]} \\ &\quad \times \left\{ (g^{\mu 1} g^{\nu 2} - g^{\mu 2} g^{\nu 1}) \epsilon^{\mu' \nu' \alpha \beta} k_\alpha p_\beta \tilde{P}_{\mu\mu'}(k) P_{\nu\nu'}(k) \right. \\ &\quad \left. + (g^{\mu 1} g^{\nu 2} - g^{\mu 2} g^{\nu 1}) \epsilon^{\mu' \nu' \alpha \beta} k_\alpha p_\beta P_{\mu\mu'}(k) \tilde{P}_{\nu\nu'}(k) \right\}. \end{aligned} \quad (120)$$

After some algebra, we obtain

$$\begin{aligned} T_{Gq}^A &= -4i \frac{g^2 C_F}{p \cdot n} \left\{ p_\mu n_\nu \int \frac{d^4 k}{(2\pi)^4} \frac{k^\mu k^\nu}{(k^2 + i\varepsilon)^2 [(k-p)^2 + i\varepsilon]} \right. \\ &\quad \left. - p \cdot n \int \frac{d^4 k}{(2\pi)^4} \frac{k_\perp^2}{(k^2 + i\varepsilon)^2 [(k-p)^2 + i\varepsilon]} \right\}, \end{aligned} \quad (121)$$

with $k_\perp^2 \equiv k_1^2 + k_2^2$. Evaluating its divergent part by the dimensional regularization, we get

$$T_{Gq}^A = \frac{\alpha_S}{2\pi} \frac{3}{2} C_F \frac{1}{2 - \omega}. \quad (122)$$

The contribution of the vertex V_B can similarly be calculated. It is given by

$$\begin{aligned} T_{Gq}^B &= \frac{1}{2 p \cdot n} \int \frac{d^4 k}{(2\pi)^4} \bar{u}(ps) i g \gamma_{\nu'} t^c \frac{i (\not{p} - \not{k})}{(p-k)^2 + i\varepsilon} \\ &\quad \times (-i n^\mu) (k^1 g^{\nu 2} - k^2 g^{\nu 1}) i g \gamma_{\mu'} t^c u(ps) \\ &\quad \times \frac{-i}{k^2 + i\varepsilon} P_{\mu\mu'}(k) \frac{-i}{k^2 + i\varepsilon} \tilde{P}_{\nu\nu'}(k) - (\mu \leftrightarrow \nu). \end{aligned} \quad (123)$$

One finds that this term vanishes because of the following property of the general axial-gauge type projection operator

$$n^\mu P_{\mu\mu'}(k) = 0. \quad (124)$$

Clearly, the vertex V_C does not contribute to $\Delta\gamma_{Gq}$ at the 1-loop order. In this way, we arrive at the standardly-known answer for $\Delta\gamma_{Gq}$ given by

$$\Delta\gamma_{Gq} = \frac{\alpha_S}{2\pi} \frac{3}{2} C_F. \quad (125)$$

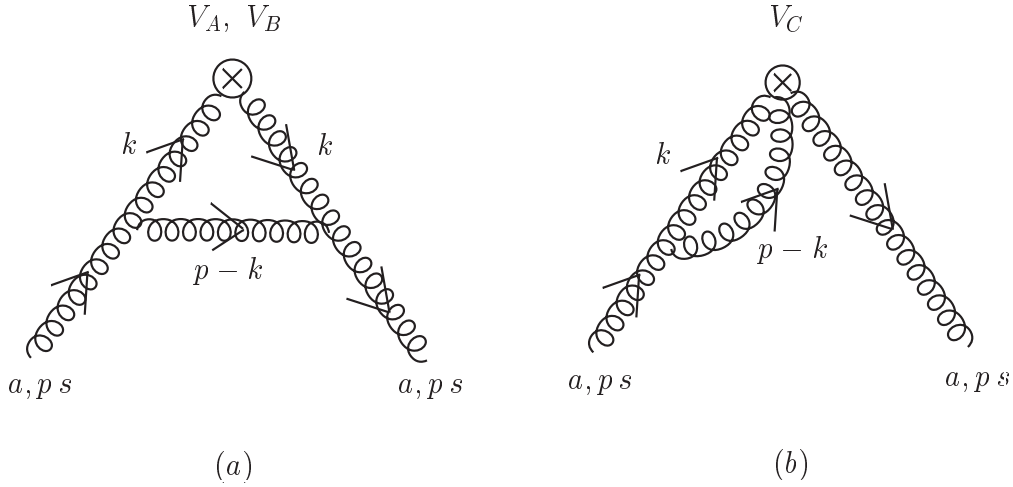


FIG. 5. The Feynman diagrams contributing to $\Delta\gamma_{GG}$.

Now we are in a position to investigate the most nontrivial part of our analysis, i.e. the anomalous dimension $\Delta\gamma_{GG}$. The contribution of the vertex V_A is given by the Feynman diagram illustrated in Fig.5(a). This gives

$$\begin{aligned} T_{GG}^A &= \frac{1}{2p \cdot n} \int \frac{d^4 k}{(2\pi)^4} \epsilon^{\lambda*}(ps) \epsilon^\rho(ps) \\ &\times g f^{ac'e} [-(p+k)_\sigma g_{\lambda\nu'} + (2k-p)_\lambda g_{\sigma\nu'} + (2p-k)_{\nu'} g_{\lambda\sigma}] \\ &\times i(k \cdot n) [g^{\mu 1} \tilde{P}^{\nu 2}(k) - g^{\mu 2} \tilde{P}^{\nu 1}(k)] \delta^{bc} \\ &\times g f^{ab'd} [(p+k)_\tau g_{\rho\mu'} + (p-2k)_\rho g_{\mu'\tau} + (k-2p)_{\mu'} g_{\rho\tau}] \\ &\times \delta^{cc'} D^{\nu\nu'}(k) \delta^{bb'} D^{\mu\mu'}(k) \delta^{de} D^{\tau\sigma}(p-k) - (\mu \leftrightarrow \nu). \end{aligned} \quad (126)$$

This can be rewritten in the form :

$$\begin{aligned}
T_{GG}^A = & + \frac{g^2 C_A}{2 p \cdot n} \int \frac{d^4 k}{(2 \pi)^4} \frac{k \cdot n}{(k^2 + i \varepsilon)^2 [(p - k)^2 + i \varepsilon]} \epsilon^{\lambda*}(ps) \epsilon^\rho(ps) \\
& \times [(p + k)_\sigma g_{\lambda\nu'} + (p - 2k)_\lambda g_{\sigma\nu'} + (k - 2p)_{\nu'} g_{\lambda\sigma}] \\
& \times [(p + k)_\tau g_{\rho\mu'} + (p - 2k)_\rho g_{\mu'\tau} + (k - 2p)_{\mu'} g_{\rho\tau}] \\
& \times (g^{\mu 1} g^{\nu 2} - g^{\mu 2} g^{\nu 1}) \tilde{P}_\nu{}^{\nu'}(k) P_\mu{}^{\mu'}(k) P^{\tau\sigma}(p - k) \\
& - \frac{g^2 C_A}{2 p \cdot n} \int \frac{d^4 k}{(2 \pi)^4} \frac{k \cdot n}{(k^2 + i \varepsilon)^2 [(p - k)^2 + i \varepsilon]} \epsilon^{\lambda*}(ps) \epsilon^\rho(ps) \\
& \times [(p + k)_\sigma g_{\lambda\nu'} + (p - 2k)_\lambda g_{\sigma\nu'} + (k - 2p)_{\nu'} g_{\lambda\sigma}] \\
& \times [(p + k)_\tau g_{\rho\mu'} + (p - 2k)_\rho g_{\mu'\tau} + (k - 2p)_{\mu'} g_{\rho\tau}] \\
& \times (g^{\nu 1} g^{\mu 2} - g^{\nu 2} g^{\mu 1}) P_\nu{}^{\nu'}(k) \tilde{P}_\mu{}^{\mu'}(k) P^{\tau\sigma}(p - k), \tag{127}
\end{aligned}$$

where $C_A = f^{abc} f^{abc} = 3$ is the standard color factor. After tedious but straightforward algebra, T_{GG}^A can further be rewritten in the form,

$$\begin{aligned}
T_{GG}^A = & + C_A \frac{g^2}{p \cdot n} \int \frac{d^4 k}{(2 \pi)^4} \frac{k \cdot n}{(k^2 + i \varepsilon) [(k - p)^2 + i \varepsilon]} \epsilon^{\lambda*}(ps) \epsilon^\rho(ps) \\
& \times \left\{ (g^{\mu' 1} g^{\nu' 2} - g^{\mu' 2} g^{\nu' 1}) \right. \\
& - (k^1 g^{\nu' 2} - k^2 g^{\nu' 1}) \frac{n^{\mu'}}{k \cdot n} + (k^1 g^{\mu' 2} - k^2 g^{\mu' 1}) \frac{n^{\nu'}}{k \cdot n} \\
& + \frac{1}{2} (k^1 g^{\nu' 2} - k^2 g^{\nu' 1}) \frac{n^2 k^{\mu'}}{(k \cdot n)^2} - \frac{1}{2} (k^1 g^{\mu' 2} - k^2 g^{\mu' 1}) \frac{n^2 k^{\nu'}}{(k \cdot n)^2} \left. \right\} \\
& \times [\epsilon_{\nu'}^* (p + k)_\sigma - 2 \epsilon^* \cdot k g_{\sigma\nu'} + \epsilon_\sigma^* (k - 2p)_{\nu'}] \\
& \times [\epsilon_{\mu'} (p + k)_\tau - 2 \epsilon \cdot k g_{\tau\mu'} + \epsilon_\tau (k - 2p)_{\mu'}] \\
& \times \left[g^{\tau\sigma} - \frac{(k - p)^\tau n^\sigma + n^\tau (k - p)^\sigma}{(k - p) \cdot n} + \frac{n^2 (k - p)^\tau (k - p)^\sigma}{[(k - p) \cdot n]^2} \right]. \tag{128}
\end{aligned}$$

We shall again calculate the three contributions from $P^{\tau\sigma}(k)$ separately. The part containing the tensor $g^{\tau\sigma}$ reduces to

$$T_{GG}^A(g^{\tau\sigma}) = T_{GG}^{A_1}(g^{\tau\sigma}) + T_{GG}^{A_2}(g^{\tau\sigma}), \tag{129}$$

where

$$T_{GG}^{A_1}(g^{\tau\sigma}) = -i C_A \frac{g^2}{p \cdot n} \int \frac{d^4 k}{(2 \pi)^4} \frac{k \cdot n (p + k)^2 - 8 p \cdot n k_\perp^2}{(k^2 + i \varepsilon) [(k - p)^2 + i \varepsilon]}, \tag{130}$$

and

$$T_{GG}^{A_2}(g^{\tau\sigma}) = +i C_A \frac{g^2}{p \cdot n} n^2 \int \frac{d^4 k}{(2 \pi)^4} \frac{k_\perp^2 (k^2 - 3 p \cdot k)}{(k^2 + i \varepsilon)^2 [(k - p)^2 + i \varepsilon] (k \cdot n)}. \tag{131}$$

The 1st part, which does not contain the $1 / (k \cdot n)$ type spurious singularity can be calculated in a standard manner, which gives

$$T_{GG}^{A_1}(g^{\tau\sigma}) = + \frac{\alpha_S}{2\pi} \frac{5}{2} C_A \frac{1}{2-\omega}. \quad (132)$$

The 2nd part can be evaluated by using the formulas :

$$\text{div} \int d^{2\omega} k \frac{k_{\perp}^2}{(k^2 + i\varepsilon) [(k-p)^2 + i\varepsilon] [k \cdot n]} = -\frac{1}{D} \left(p \cdot n^* - \frac{n^{*2}}{n^* \cdot n + D} p \cdot n \right) \bar{I}, \quad (133)$$

$$\text{div} \int d^{2\omega} k \frac{k^{\mu} k_{\perp}^2}{(k^2 + i\varepsilon) [(k-p)^2 + i\varepsilon] [k \cdot n]} = -\frac{1}{2D} \left(n^{*\mu} - \frac{n^{*2}}{n^* \cdot n + D} n^{\mu} \right) \bar{I}. \quad (134)$$

The answer is given as

$$T_{GG}^{A_2}(g^{\tau\sigma}) = -\frac{\alpha_S}{2\pi} \frac{1}{4} C_A \frac{n^2}{D} \frac{1}{[p \cdot n]} \left(p \cdot n^* - \frac{n^{*2}}{n^* \cdot n + D} p \cdot n \right) \frac{1}{2-\omega} \quad (135)$$

Collecting the two pieces, we thus arrive at

$$\begin{aligned} T_{GG}^A(g^{\tau\sigma}) = & + \frac{\alpha_S}{2\pi} \frac{5}{2} C_A \frac{1}{2-\omega} \\ & - \frac{\alpha_S}{2\pi} \frac{1}{4} C_A \frac{n^2}{D} \frac{1}{[p \cdot n]} \left(p \cdot n^* - \frac{n^{*2}}{n^* \cdot n + D} p \cdot n \right) \frac{1}{2-\omega}. \end{aligned} \quad (136)$$

Next, we evaluate the term containing the spurious singularity of $1 / (k-p) \cdot n$ in $P^{\tau\sigma}(k-p)$.

After lengthy algebra, we obtain

$$\begin{aligned} T_{GG}^A(1 / (k-p) \cdot n) = & -i C_A \frac{g^2}{p \cdot n} \\ & \times \left\{ -2 \int \frac{d^4 k}{(2, \pi)^4} \frac{k \cdot n}{(k^2 + i\varepsilon) [(k-p)^2 + i\varepsilon]} \right. \\ & - 4 p \cdot n \int \frac{d^4 k}{(2, \pi)^4} \frac{k \cdot n}{(k^2 + i\varepsilon) [(k-p)^2 + i\varepsilon] (k-p) \cdot n} \\ & + 2 n^2 \int \frac{d^4 k}{(2, \pi)^4} \frac{k_{\perp}^2}{(k^2 + i\varepsilon) [(k-p)^2 + i\varepsilon] (k-p) \cdot n} \\ & \left. - \frac{1}{2} n^2 \int \frac{d^4 k}{(2, \pi)^4} \frac{k_{\perp}^2}{(k^2 + i\varepsilon) [(k-p)^2 + i\varepsilon] k \cdot n} \right\}. \end{aligned} \quad (137)$$

Using the known integral formulas,

$$\text{div} \int d^{2\omega} k \frac{k \cdot n}{(k^2 + i\varepsilon) [(k-p)^2 + i\varepsilon] [(k-p) \cdot n]} = \bar{I}, \quad (138)$$

$$\text{div} \int d^{2\omega} k \frac{k_{\perp}^2}{(k^2 + i\varepsilon) [(k-p)^2 + i\varepsilon] [(k-p) \cdot n]} = \frac{1}{D} \left(p \cdot n^* - \frac{n^{*2}}{n^* \cdot n + D} p \cdot n \right) \bar{I}, \quad (139)$$

$$\text{div} \int d^{2\omega} k \frac{k_{\perp}^2}{(k^2 + i\varepsilon) [(k-p)^2 + i\varepsilon] [k \cdot n]} = -\frac{1}{D} \left(p \cdot n^* - \frac{n^{*2}}{n^* \cdot n + D} p \cdot n \right) \bar{I}, \quad (140)$$

we find that

$$T_{GG}^A (1 / (k - p) \cdot n) = \frac{\alpha_S}{2\pi} \left(-\frac{5}{2} C_A \right) \frac{1}{2 - \omega} + \frac{\alpha_S}{2\pi} \frac{5}{4} C_A \frac{n^2}{D} \frac{1}{[p \cdot n]} \left(p \cdot n^* - \frac{n^{*2}}{n^* \cdot n + D} p \cdot n \right) \frac{1}{2 - \omega}. \quad (141)$$

Finally, we evaluate the contribution of spurious double pole term $1 / [(k - p) \cdot n]^2$ in $P^{\tau\sigma}(k - p)$. After some algebra, we obtain

$$T_{GG}^A (1 / [(k - p) \cdot n]^2) = -i C_A \frac{g^2}{p \cdot n} n^2 \int \frac{d^4 k}{(2\pi)^4} \frac{k \cdot n}{[(k - p)^2 + i\varepsilon][(k - p) \cdot n]^2}. \quad (142)$$

Now, by using the integral formula

$$\text{div} \int d^2\omega k \frac{k^\mu}{[(k - p)^2 + i\varepsilon][(k - p) \cdot n]^2} = p^\mu \frac{2}{D} \frac{n^{*2}}{n^* \cdot n + D} \bar{I}, \quad (143)$$

we obtain

$$T_{GG}^A (1 / [(k - p) \cdot n]^2) = \frac{\alpha_S}{2\pi} C_A \frac{1}{D} \frac{n^2 n^{*2}}{n^* \cdot n + D} \frac{1}{2 - \omega}. \quad (144)$$

Summing up the three contributions, we finally arrive at

$$T_{GG}^A = \frac{\alpha_S}{2\pi} C_A \frac{n^2}{D} \frac{p \cdot n^*}{[p \cdot n]} \frac{1}{2 - \omega}. \quad (145)$$

The contribution of the vertex V_B can be calculated in the same way as that of V_A . We find that

$$\begin{aligned} T_{GG}^B = & -C_A \frac{g^2}{2p \cdot n} \int \frac{d^4 k}{(2\pi)^4} \frac{1}{(k^2 + i\varepsilon)^2 [(k - p)^2 + i\varepsilon]} \epsilon^{\lambda*}(ps) \epsilon^\rho(ps) \\ & \times [(p + k)_\sigma g_{\lambda\nu'} + (p - 2k)_\lambda g_{\sigma\nu'} + (k - 2p)_{\nu'} g_{\lambda\sigma}] \\ & \times [(p + k)_\tau g_{\rho\mu'} + (p - 2k)_\rho g_{\mu'\tau} + (k - 2p)_{\mu'} g_{\rho\tau}] \\ & \times n^\mu (k^1 g^{\nu 2} - k^2 g^{\nu 1}) \tilde{P}_\nu^{\nu'}(k) P_\mu^{\mu'}(k) P_\sigma^\tau(p - k), \\ & -C_A \frac{g^2}{2p \cdot n} \int \frac{d^4 k}{(2\pi)^4} \frac{1}{(k^2 + i\varepsilon)^2 [(k - p)^2 + i\varepsilon]} \epsilon^{\lambda*}(ps) \epsilon^\rho(ps) \\ & \times [(p + k)_\sigma g_{\lambda\nu'} + (p - 2k)_\lambda g_{\sigma\nu'} + (k - 2p)_{\nu'} g_{\lambda\sigma}] \\ & \times [(p + k)_\tau g_{\rho\mu'} + (p - 2k)_\rho g_{\mu'\tau} + (k - 2p)_{\mu'} g_{\rho\tau}] \\ & \times n^\nu (k^1 g^{\mu 2} - k^2 g^{\mu 1}) P_\nu^{\nu'}(k) \tilde{P}_\mu^{\mu'}(k) P_\sigma^\tau(p - k). \end{aligned} \quad (146)$$

One can easily convince that this term vanishes identically owing to the relations,

$$n^\mu P_\mu^{\mu'}(k) = 0, \quad n^\nu P_\nu^{\nu'}(k) = 0. \quad (147)$$

Thus we find that

$$T_{GG}^B = 0. \quad (148)$$

The contribution of the vertex V_C is given by the Feynman diagram illustrated in Fig.5(b).

This gives

$$\begin{aligned} T_{GG}^C = & \frac{1}{2p \cdot n} \int \frac{d^4 k}{(2\pi)^4} \frac{1}{(k^2 + i\varepsilon) [(p-k)^2 + i\varepsilon]} \epsilon_\lambda^*(ps) \epsilon^\rho(ps) \\ & \times [g f_{abc} n^\lambda (\tilde{P}^{1\mu}(k) g^{\nu 2} - \tilde{P}^{2\mu}(k) g^{\nu 1} + g^{\mu 1} \tilde{P}^{2\nu}(p-k) - g^{\mu 2} \tilde{P}^{1\nu}(p-k)) \\ & + g f_{ab'c'} n^\nu (\tilde{P}^{\lambda 1}(p) g^{\mu 2} - \tilde{P}^{\lambda 2}(p) g^{\mu 1} + g^{\lambda 1} \tilde{P}^{\mu 2}(k) - g^{\lambda 2} \tilde{P}^{\mu 1}(k)) \\ & + g f_{ab'c'} n^\mu (\tilde{P}^{\nu 1}(p-k) g^{\lambda 2} - \tilde{P}^{\nu 2}(p-k) g^{\lambda 1} + g^{\nu 1} \tilde{P}^{\lambda 2}(p) - g^{\nu 2} \tilde{P}^{\lambda 1}(p))] \\ & \times \delta^{bb'} D_{\mu}{}^{\mu'}(k) \delta^{cc'} D_{\nu}{}^{\nu'}(p-k) \\ & \times g f^{abc} [(p+k)_{\nu'} g_{\rho\mu'} + (p-2k)_\rho g_{\nu'\mu'} + (k-2p)_{\mu'} g_{\rho\nu'}]. \end{aligned} \quad (149)$$

This can be rewritten as

$$\begin{aligned} T_{GG}^C = & -\frac{1}{2p \cdot n} \int \frac{d^4 k}{(2\pi)^4} \frac{1}{(k^2 + i\varepsilon) [(p-k)^2 + i\varepsilon]} \epsilon_\lambda^*(ps) \epsilon^\rho(ps) \\ & \times \left\{ n^\lambda [(g^{1\mu} g^{\nu 2} - g^{2\mu} g^{\nu 1}) \tilde{P}_\mu{}^{\mu'}(k) P_{\nu}{}^{\nu'}(p-k) \right. \\ & \quad + (g^{1\mu} g^{\nu 2} - g^{2\mu} g^{\nu 1}) P_\mu{}^{\mu'}(k) \tilde{P}_\nu{}^{\nu'}(p-k)] \\ & + n^\nu [(g^{\lambda 1} g^{\mu 2} - g^{\lambda 2} g^{\mu 1}) P_\mu{}^{\mu'}(k) P_{\nu}{}^{\nu'}(p-k) \\ & \quad + (g^{\lambda 1} g^{\mu 2} - g^{\lambda 2} g^{\mu 1}) \tilde{P}_\mu{}^{\mu'}(k) P_{\nu}{}^{\nu'}(p-k)] \\ & + n^\mu [(g^{\nu 1} g^{\lambda 2} - g^{\nu 2} g^{\lambda 1}) P_\mu{}^{\mu'}(k) \tilde{P}_\nu{}^{\nu'}(p-k) \\ & \quad + (g^{\nu 1} g^{\lambda 2} - g^{\nu 2} g^{\lambda 1}) P_\mu{}^{\mu'}(k) P_{\nu}{}^{\nu'}(p-k)] \left. \right\} \\ & \times [(p+k)_{\nu'} g_{\rho\mu'} + (p-2k)_\rho g_{\nu'\mu'} + (k-2p)_{\mu'} g_{\rho\nu'}]. \end{aligned} \quad (150)$$

Here, we notice that the following relation holds

$$\epsilon_\lambda^*(ps) n^\lambda = 0, \quad \text{for } s = \pm 1, \quad (151)$$

since the external gluon is on the mass-shell with the longitudinal polarization $s = \pm 1$. Then, the 1st term in the brackets, which contains the factor n^λ , vanishes. The 2nd and 3rd terms also vanish because of the relations :

$$n^\nu P_{\nu}{}^{\nu'}(p-k) = 0, \quad n^\mu P_{\mu}{}^{\mu'}(k) = 0. \quad (152)$$

As a consequence, the contribution of the vertex V_C in the gluon spin operator vanishes, i.e.

$$T_{GG}^C = 0. \quad (153)$$

Summing up the three contributions T_{GG}^A, T_{GG}^B and T_{GG}^C , we therefore get

$$T_{GG} = \frac{\alpha_S}{2\pi} C_A \frac{n^2}{D} \frac{p \cdot n^*}{[p \cdot n]} \frac{1}{2 - \omega}. \quad (154)$$

Again, it is instructive to consider several limiting cases. In the light-cone limit with $n^0 = n^3 = 1/\sqrt{2}$ and $n^2 = 0$, one sees that T_{GG} above vanishes. This is consistent with the direct calculation in the light-cone gauge [38]. On the other hand, in the temporal limit with $n^0 = 1, n^3 = 0$ and $n^2 = 1$, the coefficient of $1/(2 - \omega)$ *diverges*, since $D \rightarrow 0$ in this limit. However, for obtaining the anomalous dimension $\Delta\gamma_{GG}$, we must also take account of the self-energy insertion to the external gluon lines. The contribution of these diagrams turn out to be (see, for instance, [54],[56])

$$T_{GG}^{Self} = \frac{\alpha_S}{2\pi} \left(\frac{11}{6} C_A - \frac{1}{3} n_f \right) \frac{1}{2 - \omega} - \frac{\alpha_S}{2\pi} C_A \frac{n^2}{D} \frac{p \cdot n^*}{[p \cdot n]} \frac{1}{2 - \omega}. \quad (155)$$

One finds that the dangerous terms in T_{GG} and T_{GG}^{Self} cancel exactly, thereby being led to

$$T_{GG} + T_{GG}^{Self} = \frac{\alpha_S}{2\pi} \left(\frac{11}{6} C_A - \frac{1}{3} n_f \right) \frac{1}{2 - \omega}, \quad (156)$$

which gives

$$\Delta\gamma_{GG} = \frac{\alpha_S}{2\pi} \left(\frac{11}{6} C_A - \frac{1}{3} n_f \right) \quad (157)$$

In this way, we have succeeded in reproducing the well-known answer completely independently of the choice of the 4-vector n^μ , which is thought to characterize the Lorentz frame in which the transverse-longitudinal decomposition is carried out and simultaneously gauge-fixing condition is imposed. The flexibility of our treatment on the choice of the 4-vector n^μ enables us to handle several interesting cases in a unified way with the help of the generalized n_μ^* -prescription. They include the temporal gauge limit with $n^2 = 1$, the light-cone gauge limit with $n^2 = 0$, and also the spatial axial-gauge limit with $n^2 = -1$, etc. We have shown that the temporal gauge limit should be treated with special care, because singular terms appear in the course of manipulation. Nevertheless, after summing up all the relevant contributions, dangerous singular terms cancel among themselves and the final answer is shown to be the same in all the cases. This gives a strong evidence to the gauge-invariance of the gluon spin in the longitudinal nucleon spin sum rule. It should be emphasized that it is just traditional gauge-invariance in the sense that the answer is exactly the same also in other gauges than the light-cone gauge.

IV. REMARKS ON THE GAUGE-INVARIANT-EXTENSION APPROACH

In [17], we have shown that the quark and gluon dynamical OAMs appearing in our nucleon spin decomposition (I) can be related to the difference between the 2nd moment of the unpolarized GPDs and the 1st moment of the longitudinally polarized PDFs as

$$\begin{aligned} L_q &= \langle ps | n_\lambda M_{q-OAM}^{\lambda 12} | ps \rangle / (n \cdot p) \\ &= \frac{1}{2} \int x [H^q(x, 0, 0) + E^q(x, 0, 0)] dx - \frac{1}{2} \int \Delta q(x) dx, \end{aligned} \quad (158)$$

and

$$\begin{aligned} L_G &= \langle ps | n_\lambda M_{G-OAM}^{\lambda 12} | ps \rangle / (n \cdot p) \\ &= \frac{1}{2} \int x [H^g(x, 0, 0) + E^g(x, 0, 0)] dx - \int \Delta g(x) dx. \end{aligned} \quad (159)$$

We think it instructive to reconsider these relations in the context of gauge-invariant-extension approach using gauge link or Wilson line. It is widely accepted that the gauge-invariant definitions of the GPDs as well as the polarized PDFs necessarily require the gauge link connecting two different space-time point. However, the quantities appearing in the r.h.s. of the above relations are not GPDs and PDFs themselves but their lower moments. In fact, the above relations can also be expressed as [4]

$$L_q = \frac{1}{2} [A_{20}^q(0) + B_{20}^q(0)] - \frac{1}{2} a^q(0), \quad (160)$$

$$L_G = \frac{1}{2} [A_{20}^G(0) + B_{20}^G(0)] - a^G(0). \quad (161)$$

Here, $A_{20}^q(0)$, $B_{20}^q(0)$, $A_{20}^G(0)$ and $B_{20}^G(0)$ are the forward limit ($t \rightarrow 0$) of the gravitational form factors $A_{20}^q(t)$, $B_{20}^q(t)$, $A_{20}^G(t)$ and $B_{20}^G(t)$, while $a^q(0)$ and $a^G(0)$ are the axial charges of quarks and gluons corresponding to the forward limits of axial form factors $a^q(t)$ and $a^G(t)$. (We recall that the quark and gluon axial charges are identified with the quark and gluon intrinsic spins in the gauge-invariant \overline{MS} regularization scheme, i.e. $a^q(0) = \Delta\Sigma$ and $a^G(0) = \Delta G$.) Note that, to extract the form factors, deep-inelastic-scattering measurements are not mandatory. For example, the gravitational form factors can in principle be extracted from graviton-nucleon elastic scattering just as the electromagnetic form factors can be extracted from electron-nucleon elastic scatterings, even though this is just a Gedanken experiment. This means that, at least for these quantities, i.e. for the form factors, we do not need to stick to such an idea that the path of gauge-link has a physical meaning as claimed in

gauge-invariant-extension approach. In fact, remember that our demonstration of the gauge-invariance of the evolution matrix for quark and gluon spins never uses such a concept like non-local gauge link connecting two different space-time points. This indicates that at least the above relations (158) and (159) are not affected by continuous deformation of the path of Wilson lines used in the definitions of the GPDs and the polarized PDFs. We therefore believe that the longitudinal nucleon spin decomposition can be made independently of the notion of “path”.

Unfortunately, the situation for another gauge-invariant decomposition (II) is not very clear yet. The reason is that the generalized canonical OAMs appearing in this decomposition can only be related to generalized transverse momentum distributions also called Wigner distributions. Recently, an interesting and useful relation between the OAMs and Wigner distributions was suggested by Lorcé and Pasquini [58]. However, gauge-invariant definition of Wigner distribution requires gauge link or Wilson line, which is generally path-dependent. Hatta showed that the LC-like path choice gives “canonical” OAM [29]. On the other hand, Ji, Xiong, and Yuan argued that the straight path connecting the relevant two space-time points gives “dynamical” OAM [33]. Assuming that both are correct, one might be lead to two possible scenarios. The 1st possibility is that, because there are infinitely many paths connecting the two relevant space-time points appearing in the gauge-invariant definition of Wigner distribution, there are infinitely many Wigner distributions and consequently infinitely many quark and gluon OAMs. This reminds us of a similar possibility indicated by Lorcé’s argument of Stueckelberg symmetry [30],[31], which however seems to be denied by our discussion given in sect.II. The 2nd possibility is that the Wigner distributions with infinitely many paths of gauge-link are classified into some discrete pieces or equivalent classes, which cannot be continuously deformable into each other. The recent consideration by Burkardt may be thought of as an indication of this 2nd possibility [59].

The idea of gauge-link is of more general concerns and has a long history. Once, DeWitt tried to formulate the quantum electrodynamics in a gauge-invariant way, i.e. without introducing gauge-dependent potential [60]. However, it was recognized soon that, although the framework is manifestly gauge-invariant by construction it *does* depend of the choice of path defining the gauge-invariant potential [61] -[64]. Since the problem seems to be intimately connected with the one we are confronting with, we think it instructive to briefly review this framework by paying attention to its subtle nature. We first recall that the QED

lagrangian has an invariance under general gauge transformation of electron field and the electromagnetic potential given as

$$\psi(x) \rightarrow \psi'(x) = e^{ie\Lambda(x)} \psi(x), \quad (162)$$

$$A_\mu(x) \rightarrow A'_\mu(x) = A_\mu(x) + \partial_\mu \Lambda(x). \quad (163)$$

As a gauge-transformation function $\Lambda(x)$, DeWitt chose as

$$\Lambda(x) = - \int_{-\infty}^0 A_\sigma(z) \frac{\partial z^\sigma}{\partial \xi} d\xi, \quad (164)$$

where $z^\mu(x, \xi)$ is a point on the line toward x , with ξ being a parameter chosen in such a way that

$$z^\mu(x, 0) = x^\mu, \quad \text{and} \quad z^\mu(x, -\infty) = \text{spatial infinity}. \quad (165)$$

Note here that $\partial z^\mu / \partial x^\lambda = \delta_\lambda^\mu$ at $\xi = 0$.

One can show that the new electron and photon fields defined through this $\Lambda(x)$ are manifestly gauge-invariant. In fact, under an arbitrary gauge transformation

$$\psi(x) \rightarrow \psi'(x) = e^{ie\omega(x)} \psi(x), \quad (166)$$

$$A_\mu(x) \rightarrow A'_\mu(x) = A_\mu(x) + \partial_\mu \omega(x), \quad (167)$$

the function $\Lambda(x)$ transforms as

$$\begin{aligned} \Lambda(x) &\rightarrow - \int_{-\infty}^0 (A_\sigma(z) + \partial_\sigma \omega(z)) \frac{\partial z^\sigma}{\partial \xi} d\xi \\ &= - \int_{-\infty}^0 A_\sigma(z) \frac{\partial z^\sigma}{\partial \xi} d\xi - \int_{-\infty}^0 \frac{\partial \omega(z)}{\partial \xi} d\xi = \Lambda(x) - \omega(x). \end{aligned} \quad (168)$$

This means that $\psi'(x)$ transforms as

$$\begin{aligned} \psi'(x) &\rightarrow e^{ie(\Lambda(x)-\omega(x))} e^{ie\omega(x)} \psi(x) \\ &= e^{ie\Lambda(x)} \psi(x) = \psi'(x), \end{aligned} \quad (169)$$

that is, $\psi'(x)$ is gauge-invariant. The gauge-invariance of $A'_\mu(x)$ can also be easily proved.

For instructive purpose, we reproduce here the proof. The manipulation goes as follows :

$$\begin{aligned}
A'_\mu(x) &= A_\mu(x) + \partial\Lambda(x) \\
&= A_\mu - \partial_\mu \int_{-\infty}^0 A_\sigma(z) \frac{\partial z^\sigma}{\partial \xi} d\xi \\
&= A_\mu - \int_{-\infty}^0 \partial_\nu A_\sigma(z) \frac{\partial z^\nu}{\partial x^\mu} d\xi - \int_{-\infty}^0 A_\sigma(z) \frac{\partial}{\partial \xi} \left(\frac{\partial z^\sigma}{\partial x^\mu} \right) d\xi \\
&= A_\mu - \int_{-\infty}^0 \partial_\nu A_\sigma(z) \frac{\partial z^\nu}{\partial x^\mu} \frac{\partial z^\sigma}{\partial \xi} d\xi \\
&\quad + \int_{-\infty}^0 \partial_\nu A_\sigma(z) \frac{\partial z^\nu}{\partial \xi} \frac{\partial z^\sigma}{\partial x^\mu} d\xi - A_\sigma(z) \frac{\partial z^\sigma}{\partial \xi} \Big|_{\xi=-\infty}^{\xi=0} \\
&= A_\mu - \int_{-\infty}^0 \partial_\nu A_\sigma(z) \frac{\partial z^\nu}{\partial x^\mu} \frac{\partial z^\sigma}{\partial \xi} d\xi \\
&\quad + \int_{-\infty}^0 \partial_\nu A_\sigma(z) \frac{\partial z^\nu}{\partial \xi} \frac{\partial z^\sigma}{\partial x^\mu} d\xi - A_\sigma(z) \delta_\mu^\sigma \\
&= - \int_{-\infty}^0 (\partial_\nu A_\sigma - \partial_\sigma A_\nu) \frac{\partial z^\nu}{\partial x^\mu} \frac{\partial z^\sigma}{\partial \xi} d\xi.
\end{aligned} \tag{170}$$

We thus find the key relation

$$A'_\mu(x) = - \int_{-\infty}^0 F_{\nu\sigma}(z) \frac{\partial z^\nu}{\partial x^\mu} \frac{\partial z^\sigma}{\partial \xi} d\xi. \tag{171}$$

Since the r.h.s. of the above relation is expressed only in terms of gauge-invariant field-strength tensor, the gauge-invariance of $A'_\mu(x)$ is obvious. This is the essence of the gauge-invariant formulation of QED by DeWitt. Here is a catch, however. Although the r.h.s. of (171) is certainly gauge-invariant, it generally depends on the path connecting the point x and spatial infinity. To see it most transparently, let us take constant-time paths in a given Lorentz frame, with the property $\partial z^0 / \partial \xi = 0$. In this case, Eq. (164) reduces to

$$\Lambda(x) = - \int_{-\infty}^x \mathbf{A}(x^0, \mathbf{z}) \cdot d\mathbf{z}. \tag{172}$$

Let us now consider two space-like (or constant-time) paths L_1 and L_2 connecting x and spatial infinity [61]. The corresponding gauge-invariant electron fields are given by

$$\psi'(x; L_1) = \exp \left[-i e \int_{L_1}^x \mathbf{A}(x^0, \mathbf{z}) \cdot d\mathbf{z} \right] \psi(x), \tag{173}$$

$$\psi'(x; L_2) = \exp \left[-i e \int_{L_2}^x \mathbf{A}(x^0, \mathbf{z}) \cdot d\mathbf{z} \right] \psi(x). \tag{174}$$

These two gauge-invariant electron fields are related through

$$\psi'(x; L_1) = \exp \left[i e \left(\int_{L_1}^x - \int_{L_2}^x \right) \mathbf{A}(x^0, \mathbf{z}) \cdot d\mathbf{z} \right] \psi'(x; L_2). \tag{175}$$

Closing the path of integration to a loop L by a connection at spatial infinity, where all fields and potentials are assumed to vanish, we obtain

$$\begin{aligned}
\psi'(x; L_1) &= \exp \left[i e \oint_L \mathbf{A}(x^0, \mathbf{z}) \cdot d\mathbf{z} \right] \psi'(x; L_2) \\
&= \exp \left[i e \iint_S (\nabla_z \times \mathbf{A}(x^0, \mathbf{z})) \cdot d\mathbf{z} \right] \psi'(x; L_2) \\
&= \exp \left[i e \iint_S \mathbf{B}(x^0, \mathbf{z}) \cdot d\mathbf{z} \right] \psi'(x; L_2).
\end{aligned} \tag{176}$$

Since the magnetic flux will not vanish in general, $\psi'(x; L_1)$ and $\psi'(x; L_2)$ do not coincide, which means that $\psi'(x)$ is generally *path-dependent*.

Very interestingly, there is one interesting choice of $\Lambda(x)$, which enables us to construct $\psi'(x)$ and $A'_\mu(x)$, which are path-independent as well as gauge-invariant [65],[66]. The choice corresponds to taking as

$$\Lambda(x) = - \int_{-\infty}^x \mathbf{A}_\parallel(x^0, \mathbf{z}) \cdot d\mathbf{z}, \tag{177}$$

where $\mathbf{A}_\parallel(x)$ is the longitudinal component in the decomposition

$$\mathbf{A}(x) = \mathbf{A}_\perp(x) + \mathbf{A}_\parallel(x), \tag{178}$$

with the important properties

$$\nabla \cdot \mathbf{A}_\perp = 0, \quad \nabla \times \mathbf{A}_\parallel = 0. \tag{179}$$

Since

$$\oint_L \mathbf{A}_\parallel(x^0, \mathbf{z}) \cdot d\mathbf{z} = \iint_S (\nabla_z \times \mathbf{A}_\parallel(x^0, \mathbf{z})) \cdot d\mathbf{S} = 0, \tag{180}$$

due to the irrotational property of $\mathbf{A}_\parallel(x)$, the electron wave function defined by

$$\psi'(x) = \exp \left[-i e \int_{-\infty}^x \mathbf{A}_\parallel(x^0, \mathbf{z}) \cdot d\mathbf{z} \right] \psi(x) \tag{181}$$

is not only gauge-invariant but also path-independent. Remember here the fact that the transverse and longitudinal components of \mathbf{A} can be expressed as

$$\mathbf{A}_\perp(x) = \mathbf{A}(x) - \nabla \frac{1}{\nabla^2} \nabla \cdot \mathbf{A}(x), \quad \mathbf{A}_\parallel(x) = \nabla \frac{1}{\nabla^2} \nabla \cdot \mathbf{A}(x). \tag{182}$$

Therefore, $\psi'(x)$ can be reduced to the following form,

$$\begin{aligned}
\psi'(x) &= \exp \left[-i e \int_{-\infty}^x \left(\nabla_z \frac{1}{\nabla_z^2} \nabla_z \cdot \mathbf{A}(x^0, \mathbf{z}) \right) \cdot d\mathbf{z} \right] \psi(x) \\
&= \exp \left[-i e \frac{\nabla \cdot \mathbf{A}}{\nabla^2}(x) \right] \psi(x).
\end{aligned} \tag{183}$$

Note that, in this form, the path-independence of $\psi'(x)$ is manifest. This quantity is nothing but the gauge-invariant physical electron introduced by Dirac [67]. (For more discussion about it, we recommend the references [48],[49].) Using the same function $\Lambda(x)$, the gauge-invariant potential $A'_\mu(x)$ can also be readily constructed as

$$\mathbf{A}'(x) = \mathbf{A}_\perp(x), \quad (184)$$

$$A'^0(x) = A^0(x) + \int_{-\infty}^x \dot{\mathbf{A}}_\parallel(x^0, \mathbf{z}) \cdot d\mathbf{z}. \quad (185)$$

In this way, one reconfirms that the spatial component of the gauge-invariant potential $A'_\mu(x)$ is nothing but the transverse component of $\mathbf{A}(x)$. Note that it precisely coincides with the quantity projected out by the before mentioned projection operator

$$P^{\mu\nu} = g^{\mu\nu} + \frac{\partial^\mu \partial^\nu - n \cdot \partial (n^\mu \partial^\nu + n^\nu \partial^\mu) + \square n^\mu n^\nu}{(n \cdot \partial)^2 - \square}, \quad (186)$$

with $n^\mu = (1, 0, 0, 0)$. In fact, for $i, j = 1, 2, 3$, this gives

$$P^{ij} = \delta^{ij} - \frac{\nabla^i \nabla^j}{\nabla^2}. \quad (187)$$

As is clear from the discussion above, except for some fortunate choice of $\Lambda(x)$, the fields $\psi'(x)$ and $A'_\mu(x)$ defined by (162) and (163) supplemented with (164) are by construction gauge-invariant but generally path-dependent. How should we interpret this path-dependence. Soon after the paper by DeWitt appeared [60], Belinfante conjectured that a path is just a “gauge” [61]. He showed that, by averaging over path-dependent potential over the directions of all straight lines at constant time converging to the point where the potential is to be calculated, one is led to the potential in the Coulomb gauge [61]. On the other hand, Rohrlich and Strocchi applied a similar averaging procedure over covariant path and they obtained the potential in the Lorentz gauge [63]. It was also demonstrated by Yang that, for a simple quantum mechanical system, the path-dependence is eventually a reflection of the gauge-dependence [64]. All these investigations appears to indicate that, if a quantity in question is seemingly gauge-invariant but path-dependent, it is not a gauge-invariant quantity in a *true* or *traditional* sense, which in turn indicates that it may not correspond to genuine observables. Clearly, the GIE approach is equivalent to the standard treatment of gauge theory, only when its extension by means of gauge link is path-independent. By the standard treatment of the gauge theory, we mean the following. Start with a gauge-invariant quantity or expression. Fix gauge according to the need of practical calculation. Answer should be independent of gauge choice.

V. CONCLUSION

We have investigated the uniqueness or non-uniqueness problem of the decomposition of the gluon field into the physical and pure-gauge components, which is the basis of the recently proposed two physically inequivalent gauge-invariant decomposition of the nucleon spin. It was emphasized that, the physical motivation of this decomposition is the familiar transverse-longitudinal decomposition in QED, which is known to be unique once the Lorentz-frame of reference is chosen. Although this transverse-longitudinal decomposition is a intrinsically a Lorentz-frame-dependent operation, by the introduction of a constant 4-vector n^μ , one can make this transverse-longitudinal decomposition into a seemingly covariant form, although at the lowest order in the gauge coupling constant in the case of nonabelian gauge theory. This in turn enables us to write down a lowest order expression of the gauge-invariant gluon spin operator usable in a desired Lorentz frame. Choosing n^μ to be a temporal vector with $n^2 > 0$, light-cone vector with $n^2 = 0$, or spatial vector with $n^2 < 0$, we are able to work in any of the temporal gauge, the light-cone gauge, or spatial axial gauge with the corresponding choice of the Lorentz-frame. We confirmed that the 1-loop evolution equations for the quark and gluon spins in the nucleon are entirely independent of the choice of n^μ , which at the least means that the light-cone gauge is not a unique or a preferential gauge for the gluon spin problem.

On the basis of these findings together with common wisdom of electrodynamics, we argued that there exist two and only two physically inequivalent gauge-invariant decompositions of the nucleon spin, in sharp contrast to the conflicting view that there are infinitely many decompositions of the nucleon spin. These two decompositions, which we call (I) and (II), are characterized by two different OAMs for quarks and gluons, i.e. the “dynamical” OAM and the generalized “canonical” OAM. We have confirmed the fact that the dynamical OAMs of quarks and gluons appearing in the decomposition (I) can in principle be extracted model-independently from combined analysis of GPD and polarized PDF measurements. This means that we have reached at least one satisfactory solution to the nucleon spin decomposition problem.

On the other hand, the observability of the OAM appearing in the decomposition (II), i.e. the generalized “canonical” OAM, is not completely clear yet. This is because, although the relation between the “canonical” OAM and a Wigner distribution is suggested,

its path-dependence or path-independence should be clarified more thoroughly. Moreover, once quantum loop effects are included, the very existence of TMDs as well as Wigner distributions satisfying gauge-invariance and factorization (or universality) at the same time is under debate. (See [68], and references therein.) Is process-independent extraction of canonical OAM possible ? This is still a challenging open question.

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